Material properties: effect on deflection, rotation and vibration — Part 1

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Introduction

Over the past 20 years, structural engineering has been subjected to greater demands due to a wider variety of materials in use, and the necessity to construct in a sustainable manner in order to obtain more energy-efficient buildings. Furthermore, when more than one material is in use, shrinkage problems become more evident, while differential corrosion of different metals in contact with each other has to be ascertained. Spans are getting longer, while new, advanced structural material is becoming lighter. It is therefore not surprising that the effect of cracking in building materials due to deflection effects, together with the problem of vibrations onto structural floors, is increasing. Some idea of this transition can be obtained by noting that whereas the sway in a strong wind of the Empire State Building is limited to 100mm, the sway of modern skyscrapers may be as high as 1000mm[1].

The serviceability aspect has thus been growing in importance and, in particular instances, may be as critical a design criterion as the strength requirement. A structure deemed to satisfy the strength requirement may be classified as offering adequate structural robustness. It can, however, fail on the 'fit-for-purpose' requirement due to deflection/vibration problems.

Further cracking is also an issue. The measurement of crack widths gives an indication of damage sustained. A Building Research Establishment publication classifies damage into six categories: 0–5 according to the existing crack width[2]. Categories 4 and 5, which outline structural damage, note crack widths as varying from 15mm to over 25mm. Category 2 damage, with crack widths up to 5mm which may be easily filled, may lead to sticking of doors or windows which requires easing and adjusting, together with lack of weather tightness. Category 0 refers to hairline cracking, where the width is less than 0.1mm, requiring no action.

The understanding of structural materials is an academic discipline in itself. In an Institution of Civil Engineers' Brunel International Lecture in 2000, Burdekin outlined the varying properties of traditional and modern materials[3]. It is important to consider the part played by the choice of materials, as these are vital to the performance and integrity of all forms of construction. Smith and Jagger[4] note that the overall cost of a project for hi-tech buildings is quoted at one-third for structure, one-third for finishings and one-third for services, with only 6% of total costs allocated for the material costs. The number of materials used by the construction industry has grown from a few hundred in the 19th century to over 160 000 today. Traditional materials are alloy steel, concrete, timber and masonry; newer materials include plastics, composite materials such as carbon fibre, glass fibre and metal matrix composites, duplex and other stainless steels, non-ferrous materials, and new varieties of additives for concretes[5].

This article is the first in a three-part series outlining the serviceability calculations to be undertaken in the preliminary structural design stage.

Here, we consider the structural properties of materials not only in terms of their strength, but also their serviceability in use. The magnitude of the serviceability characteristics, as defined by deflection and the ensuing rotation at the supports, which leads to cracking and the problematic effect of vibration, are examined via hand calculations that give an 'engineering feel' to the reader.

Two design examples then follow in Parts 2 and 3, which will demonstrate these hand calculations when undertaken in structural steel, timber, composite steel and masonry panelled construction. The first example in Part 2 will be limited to simple beam/slab span effects, while the second example (Part 3) will compare the floor slab configuration for warehouse loading in composite steel construction with prestressed hollow slabs. It will go on to note the basic hand calculation for a sidesway deflection check to a three-storey vertical structure.

Table 1: Updated C deflection coefficient for I cm⁴ calculation for a simple support span condition for UDLs and central point loads

Span-to-deflection ratio	Steel E = 205kN/mm²		Timber E = 8kN/mm²	
	UDL	Point load	UDL	Point load
1/200	1.27	2.03	43.3	52
1/360	2.29	3.66	78.0	93.75
1/500	3.17	5.08	108.0	148.63
1/800	5.08	8.12	173.0	208.08
1/1000	6.35	10.15	216.0	260.10

Hand calculations for the Serviceability Limit State

A serviceability shortfall can be gauged from a simple, shading timber structure (Figure 1). In this instance, although there are no dire consequences – and possibly even an improvement to the rainwater shedding – this could be noted as an aesthetic shortfall.

These methods are applicable to a variety of basic structures, from simple shading devices (as illustrated in Fig. 1) to basic grillages supporting building services, while also acting as aids for the preliminary design to bridge decks. The examples that follow note the hand calculations to be undertaken (which are not time-consuming), via five basic equations aided by a BCSA chart[6].

Two earlier technical papers published by the author[7,8] discuss deflection, rotation and vibration in buildings, and refer to serviceability excluding structural collapse. These note that the serviceability requirements depend on the end use of a structure, with an agricultural shed being more lenient on serviceability requirements than the non-load-bearing partitions in new residential premises.

Figure 1 Sagging 45-year-old timber rafter – or is the camber intentional for the shedding of waters?!

The methods outlined for the calculation of deflections are based on the method outlined by the BCSA in producing deflection coefficients C for steel members (Figure 2)[6].

The required moment of inertia I in cm^4 for a uniform distributed load (UDL) is obtained from Equation 1, where C is a factor obtained from Fig. 2 or Table 1, dependent on the span-to-deflection ratio adopted, w is the serviceability load in kN/m and L is the effective span in metres. The units thus employed are consistent with the value of the constant C adopted.

$$I = CwL^3 \quad (1)$$

Where serviceability is not an issue, it is recommended that the specified span-to-deflection ratio relate solely to the deflection, as calculated for the variable load $Q1$ alone, which is the imposed load. However, instances arise where it is necessary to minimise crack widths to non-load-bearing partitions which may also have cladding affixed or for vibration control; in these instances, the total serviceable load is to be applied.

For a point load equation I is modified:

$$I = CWL^2 \quad (2)$$

Again, Table 1 gives C values for central point loads, with W taken as the total serviceability load in kN. For loads other than central loads, the C values are taken from Fig. 2.

Scaling down by inverse proportion, this method is applicable for use in other structural materials such as timber. For timber, this scaling is obtained by multiplying the quotient of Young's modulus E_s for steel divided by $E_{0,mean}$ for timber. The C value calculation then calls for a refinement to cater for timber creep effects over time.

For reinforced concrete, this method is applicable only in analysing horizontal drift of a vertical structure. In this instance, the allowable drift recommended is based on span-to-deflection ratios, unlike the vertical loading situations. For vertical loads, the traditional span-to-depth ratios are applicable, although they should be applied flexibly to cater for the intended use of the structure. When designing for a simply supported roof slab to a shed, by how much can a span-to-depth ratio of 20 be exceeded, prior to the serviceability aspect becoming suspect? This will possibly also depend on the allowable falls for the disposal of run-off waters to remain unhindered.

It is even easier to go from a deflection-to-span ratio to a rotation calculation. The rotation in radians θ is given by the simple relationship:

$$\theta = 3.2\ x\ deflection\text{–}to\text{–}span\ ratio \quad (3)$$

Similarly, adopting relatively smaller deflection-to-span ratios than those normally adopted for designs based solely on deflection criteria will cater for vibration effects in steel or timber. In Part 2 of the series, a simple calculation check will be undertaken for the vibration effects as imposed on a gym floor in a light steel-and-timber construction.

Overall, excessive vibrations can be avoided by designing floor systems to have fundamental frequencies typically above 8Hz[9]. The Institution's Technical Guidance Note on 'Floor vibration'[10] notes that for office buildings the minimum frequency is 4Hz, while for stages and dance floors this minimum is increased to 8.4Hz. It is

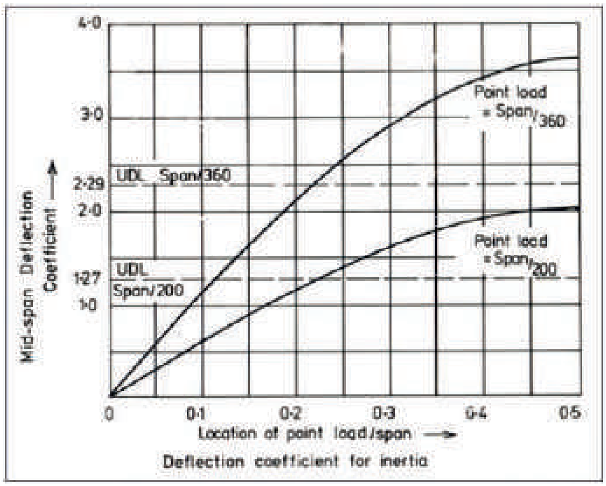

Figure 2
Deflection coefficients C for moments of inertia for steel sections

further noted that the natural frequency f_1 of a simply supported beam may also be obtained from its maximum deflection δ due to the applied load and self-weight in millimetres:

$$f_1 = 18 / \delta^{1/2} \quad (4)$$

On-site, a non-intrusive, preliminary, cheap testing plan is useful for characterising the global performance of a floor in terms of human footfall. A heel drop is generated by an 80kg person arching their heels up by 60mm on the balls of their feet and then freefalling onto the floor. The peak force is about 2.2kN and the duration of the impulse is 50ms[11]. The heel drop does not require an assessment of the flooring damping system, as the (viscous) human body absorbs mechanical energy whenever it is in contact with the floor.

EC5 notes that a residential timber floor may be considered to satisfy vibration criteria if the natural frequency of the floor exceeds 8Hz. Furthermore, the immediate deflection under a 1kN point, which represents a person walking on the floor, should not exceed the deflection δ where l is the span given in millimetres[12].

$$\delta = 16\,500 / l^{1.1} \text{ or } 18mm \text{ if } l < 4m \quad (5)$$

The term 'serviceability' refers to all structural behaviour, excluding structural collapse which renders a building or construction unfit for its intended use. This lack of fitness may relate to human reactions (aesthetic, physiological or psychological), ranging from annoyance to medical trauma. It may also hinder the operation of humans or equipment. In theory, it is possible to modify an unserviceable building, so that it becomes serviceable. Serviceability limits and performance standards as also influenced by non-structural matters such as architectural features, auditory and visual stimuli, and building usage, which militate against a single value; instead, these should be specified in terms of sets of limits[1].

Structural material properties

Table 2 outlines the material properties that provide values on the:

- **strength of the material**
- **modulus of elasticity** E which outlines the deflection characteristic of the material – the higher the E value, the lower the deflection
- **density of the material** – the weight of the building is normally greater than that of its contents; lighter materials thus call for smaller sections, although they have increased vibratory effects
- **coefficient of thermal expansion** – this defines thermal movements noted as low for timber and limestone masonry
- **embodied energy** – this relates to the sustainability of the material adopted and to the energy and resources expended in the manufacture and transport of materials. Even in a very efficient building, ongoing energy use over the lifetime of the building will represent four times that of the embodied energy used in the construction process. However, the proportion of energy that is either embodied or operational varies between types. The extremes are a bridge, with high embodied energy and low operational requirements, and a hospital, where the operational energy is high. Finally, when the building reaches the end of its life, the energy required to alter or demolish the development, and to deal with the resulting site and materials, completes the lifetime environmental costs of that development[13]. Softwood timber is the most sustainable material, with aluminium the most unsustainable
- **factor of safety** – definitions of structural materials fall under 'ductile' or 'brittle'. The various codes of practice note that ductile materials, such as structural steelwork, have a material factor of safety tending towards 1. Ductile behaviour sustains excessive local strains by plastic deformation, noted as the flat portion for steel in Figure 3, thereby giving warning before damage occurs. Concrete, on the other hand, with a material factor of safety of 1.5, appears to sit on the dividing line between ductile and brittle. Other materials with a material factor of safety higher than 1.5 are defined as brittle. Brittle materials break without advance warning in the elastic range of deformation (Fig. 3 shows the stress strain curve for glass)

Although Fig. 3 notes glass as being a very brittle material, Table 2 notes its E value and density to be similar to aluminium. Thus, although the same C factor may be utilised for calculating span-to-deflection ratios for both these materials, it is important to note that glass in panes can deflect by more than its own thickness. This takes designers into the realm of large deflection theory (Fig. 3), when the pane deflects by more than half of its thickness[14]. To avoid causing alarm, acceptable span-to-deflection ratios for glass are limited to span/65 or should not exceed 50mm[15]. In the case of prestressed glass, its strength may not be fully exploited as the deflection limit can control the design.

It is noted that the density and the modulus of elasticity E for aluminium are approximately one-third that of steel. Thus, the deflections due to dead loads for these materials are similar, even though the aluminium structure is much lighter. On the other hand, the embodied energy content identifies steelwork as being a more sustainable material, with the production of aluminium having more impact on the environment. Similarly, for higher grades of steel used at the higher stress levels, the deflections will be larger than for mild steel, as the E value is constant for different types of steelwork.

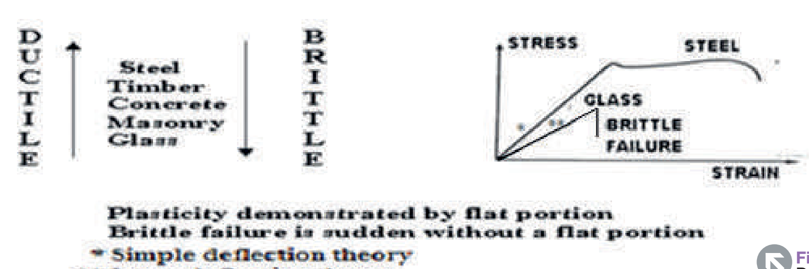

Figure 3
Ductile and brittle materials

Table 2: Comparison of properties for structural materials

Material	Ultimate stress (N/mm²)	Modulus of elasticity (N/mm²)	Density (KN/m³)	Coefficient of thermal expansion $*10^{-6}$/°C	Embodied energy (MJ/kg) (Embodied CO_2) (kg/t)	Material factor of safety (ECs & PREN) (γ_m)
Mild steel	275	205 000	70	10.8	35 (2030)	1.0
High-yield steel	460	200 000	70	10.8	35 (2030)	1.0
Prestressing wire	1570	200 000	70		35 (2030)	1.15
Aluminium alloy	255	70 000	24	23.0	300 (17 000)	1.2
Timber: Softwood / Hardwood	10–30** / 35–70**	8000** / 12 000**	6	3.5** / 3.5**	2 (1644) / 3 (2136)	1.3
Reinforced concrete	20–60	28 000 – 40 000	24	10.8	8 (203)	1.5
Glass fibre composite	250	20 000	18		100 (8070)	1.7
Limestone masonry	7.5	17 000	20	4.0	3 (2136)	2.3–3.0
Annealed glass	13 (45*)	70 000	25	8.3	15 (1130)	1.8
Prestressed glass	45 (150*)	70 000	25	8.3	20 (1130)	1.2–1.8

* Gust loading ** Parallel to grain

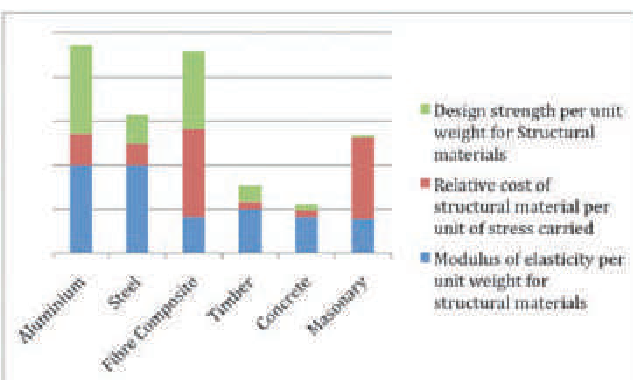

Figure 4
Comparison of structural and functional properties of various materials

As fatigue performance relates to a material's E value, the fact that aluminium's E value is only one-third that of steel means that the fatigue strength of welded aluminium alloys at a given life is about one-third that of corresponding steel joints. It is therefore important to determine the natural frequency of vibration of the component and take account of the fatigue loading to be encountered[3].

As glass fibre's E value stands at 1/10 that of steel, and it has lower bond characteristics, concrete reinforced with such elements is different from conventional concrete. Essentially, concrete reinforced with glass-reinforced plastic shows larger crack widths, larger deformations and lower shear strength than conventional concrete reinforced with the same amount of steel[3]. Again, steel reinforcing bars are more sustainable than glass fibre reinforcement.

The long-term re-use of materials after demolition of structures also needs to be accounted for. Steel is easily recycled, while concrete and masonry are easily re-used as hardcore. Only timber may be regarded as a renewable source; however, to protect the rain forests, the use of tropical hardwoods should be restricted to those obtained from properly managed schemes where timber is replaced.

Material selection

The main goal of material selection is to minimise cost while meeting product performance goals. Of course, cost per kilogram is not the only important factor in material selection. An important concept is 'cost per unit of function'. For example, if the key design objective were the stiffness of a plate of the material, then the designer would need a material with the optimal combination of density, Young's modulus and price. A valuable insight into the inherent properties of a wide range of different materials is given by the series of 'property charts' developed by Ashby[16]. These charts are plotted on logarithmic scales due to the large variation in material properties. For example, Young's modulus of elasticity E may vary by 10M times, while the density of various materials may vary by as little as 2000 times.

Figure 4 is based on values in Seward's textbook on understanding structures[17]. One of the properties is the strength of the different materials per unit of weight – known as specific strength. It is noted that aluminium and fibre composites are more competitive, with the least competitive being masonry and concrete. The weight of a building is usually greater than its contents. Constructing a lighter structure results in smaller structural members. On the other hand, weight can be useful to resist wind loads. Steelwork generally requires fireproofing or a paint treatment to offset corrosion effects, which consequently reduces its efficiency.

Deflection criteria, not strength, dictate the design of structures, especially where the higher grade of material is applied, unless short spans are involved. The stiffness per unit of weight, known as specific modulus, is a better criterion. Fig. 4 also demonstrates why steel and aluminium appear more advantageous. The previous section, however, discussed the fatigue weakness of aluminium as opposed to steelwork.

The most economical structural materials appear to be concrete, timber and steelwork. It is, however, worth noting that labour costs have been ignored in Fig. 3, while fire protection, as in steelwork, may also be required at an added cost. Although masonry appears expensive as a structural material, the cost also depends on the country where it is applied. For a country with low labour costs and restricted specialisations, it could turn out to be more economical.

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Material properties: effect on deflection, rotation and vibration — Part 2

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Introduction

This article is the second in a three-part series outlining the serviceability hand calculations to be undertaken in the preliminary structural design stage.

Structural design incorporating serviceability requirements

Further to the five basic serviceability equations outlined in Part 1[1], the following example uses the coefficients of load combinations and material characteristic strengths set out in the relevant Eurocodes. It goes beyond strength calculations to examine the effects of deflection, vibration and rotation at the supports of an element. These calculations are also useful when verifying the outputs produced from structural analysis and design programs. Although the simplifications implicit in these formulae make them ideal for hand calculations, the use of automated calculation tools should not be ruled out. Options include creating a spreadsheet for these equations or even inputting them into a programmable calculator.

In this calculation, reference is made to Table 1 for calculation of C values, and to Equation 3 in Part 1 for calculation of rotation in radians. Here, Table 1 is an updated version of the table of load values published in Part 1. Whereas Part 1 referred to a steel E value of 205kN/mm², as described by the British Constructional Steel Association in 1990[2], here we use a steel E value of 210kN/mm², as set out more recently in EC3[3]. Adopting the E values of EC3 results in minor savings in section sizings for the serviceability effects. These savings are augmented in the strength calculation undertaken, as the partial safety factors cited in EC3 are lower.

Note that the timber C values for lightweight timber floors account for a 33% increase in value over the interpolated E values of the materials, to cater for creep effects. To allow for creep, the Institution's Technical Guidance Note 'Design of timber floor joists'[4] states that the instantaneous deflection due to permanent loads is to be increased by a factor of $(1 + k_{def})$, while for imposed loads a reduced factor of $(1 + \psi_{2.1}k_{def})$ is applied. Shear deflection can also be accounted for by adding 10% to the calculated deflection.

With k_{def} given at 0.8 for an internal environment and $\psi_{2.1}$ taken at 0.3, the calculated deflection has to be increased by 33% to cater for all these effects. The effect of the distributed load on the deflection calculation is considered insignificant.

This signifies that for timber, Equations 1 and 2 in Part 1 have to be factored by 1.33, after factoring up with the quotient of Young's modulus for steel divided by that for timber. This has been accounted for when computing the timber C values (Table 1).

Reference is then made to Equation 3 in Part 1 for the calculation of rotation at the support in radians. Figure 1 highlights the relevance of this rotation at the base of a simple timber structure supported on a masonry pier. This rotation of the lightly loaded canopy also creates a vertical distortion to the masonry pier due to the sagged shape of the timber rafter, which produces a horizontal thrust on the pier.

The Institution's *Manual for the design of steelwork building structures to EC3*[5] notes the following on vibration criteria:

- the fundamental frequency of floors in dwellings and offices should not be less than three cycles/second. This may be deemed to be satisfied when the total deflection is less than 28mm
- the fundamental frequency of floors used for dancing and gymnasia should not be less than five cycles/second. This may be deemed to be satisfied when the total deflection is less than 10mm

Earthquake engineers note a rule of thumb whereby 'soft' skeleton structures have a period of fundamental natural oscillations equal to roughly 1/10 of the number of floors in seconds[6]. For less ductile structures this constant doubles to 1/5. The period of a 15-storey

Table 1: Updated deflection coefficient C for I cm⁴ calculation for a simple support span condition for UDLs and central point loads

Span-to-deflection ratio	Steel E = 210kN/mm²		Timber E = 8kN/mm²	
	UDL	Point load	UDL	Point load
1/200	1.24	1.98	43.3	52.0
1/360	2.23	3.57	77.9	93.7
1/500	3.10	4.96	108.2	130.2
1/800	4.96	7.94	173.2	208.4
1/1000	6.20	9.92	216.5	260.4

building consequently equals approx. 1.5s, mainly at 0.67Hz. For a typical site, there may be 3–4 strong responses with a frequency up to about 5Hz within a frequency range of 0.2–20Hz. The higher frequencies refer to ground motion close to the epicentre on firm soil or rock, while the lower frequencies refer to earthquake motion at some distance from the epicentre and on softer soils.

As an example, consider the deflection δ, to be limited to span/200, of a simply supported steel and timber beam with Young's modulus E = 210kN/mm^2 and 8kN/mm^2 respectively. For a simply supported beam of effective span L in mm as subjected to a uniformly distributed load (UDL) of w in kN/m, the central deflection in mm is given by:

$$\delta = \frac{L}{200} = \frac{5wL^4}{384EI}$$

$$I = \frac{5 \times 200 \times w \times L^3}{384E}$$

$$I = \frac{5 \times 200 \times w \times (L \times 1000)^3}{(384 \times E \times 1000) \times 10^4} = CwL^3$$

When using $I = CwL^3$, where w is in kN/m and L is in m, the moment of inertia I is calculated in cm^4 by dividing by 10^4. This assumes consistent units throughout in mm. In this case:

$$C = \frac{5 \times 200 \times 10^9}{(384 \times 210 \times 10^7)} = 1.24$$

where the C value of 1.24 conforms to the value quoted in Table 1.

For a timber section on a simply supported span, if we restrict the deflection to span/500:

$$C = \frac{3.10 \times 1.33 \times 210}{8} = 108.2$$

In this case, 3.10 refers to the steelwork's C value for this span-to-deflection ratio, as outlined in Table 1. The constant 1.33 represents a 33% increase for timber sections, as discussed earlier. The C value of 108.2 conforms to the value quoted in Table 1.

Structural design of intermediate light flooring

The following example is for the construction of a mezzanine intermediate floor in an existing indoor public swimming pool which is to be utilised as a public gym. The structural materials adopted will ensure that the construction does not disrupt the existing use of the premises.

Figure 1 Rotation at support of sagged 45-year-old timber rafter

The proposed intermediate floor has an effective span of 7.5m between load-bearing masonry walls, with an overall depth of 6.25m and a storey height of 3.5m. The structural grid adopted is shown in Figure 2, with secondary steel beams (B1) spanning 6.25m onto a steel main beam (B2) with a 7.5m span.

Spanning onto the secondary beams are continuous softwood C18 timber planks on a structural grid of 1.25m centres. The front open façade of this mezzanine floor is glazed in laminated glass, which reduces the risk of damage and injury at a cost not much greater than that of normal annealed glass. Laminated glass prevents glass shards from falling and flying through the air, and ensures the fenestration remains sealed, even during the most severe loading conditions.

Design of timber flooring on a 1.25m continuous span

The timber floor plank is to be a C18 grade planed softwood with a density of 3.5kN/m^3 and $E_{0,mean}$ of 8kN/mm^2. We assume a 25mm thick plank, continuous across a minimum of three spans. For this condition, a bending moment (BM) of wl^2/10 is used, instead of the simple BM of wl^2/8.

The floor loading comprises a dead load (DL) of timber planks and a gym live load (LL) which is assumed to be 5kN/m^2:

$$\begin{array}{lllll} \text{DL} = 0.025\text{m} \times 3.5\text{kN/m}^3 & \times 1 = & 0.09\text{kN/m}^2 & \times 1.35 = & 0.12\text{kN/m}^2 \\ \text{LL} = 5.0\text{kN/m}^2 & \times 1 = & \underline{5.00\text{kN/m}^2} & \times 1.50 = & \underline{7.50\text{kN/m}^2} \\ & & 5.09\text{kN/m}^2\ (\text{SLS}) & & 7.62\text{kN/m}^2\ (\text{ULS}) \end{array}$$

where the ultimate limit state (ULS) load is obtained by multiplying the serviceability limit state (SLS) load by partial safety factors defined in EC0[7].

The ultimate BM M_u is calculated as:

$$M_u = \frac{7.62 \times 1.25^2}{10} = 1.19\text{kNm/m}$$

$$M_u = \frac{f \times z}{\gamma_m} = \frac{fbd^2}{6\gamma_m}$$

Table 2: I values and relative plank depths d*			
	Strength	Deflection-to-span/250*	Deflection-to-span/150*
I (cm^4)	130.2	280	168
d (mm)	25	32.25	27.22

* These ratios relate to a simple support condition

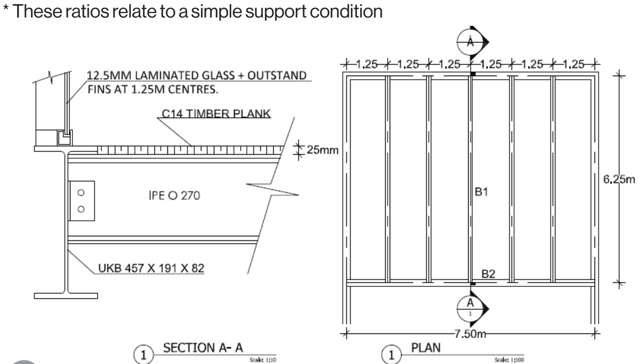

Figure 2
Details of proposed lightweight intermediate flooring

where the characteristic strength value f for C18 timber is 18N/mm^2, the breadth b is taken per metre and the partial safety factor γ_m for timber is 1.3. *Design of timber floor joists* notes that a factor k_{mod} exists on the duration of the load[4]. For intermediate floors defined as service class 1 and for permanent self-weight and medium-term imposed floor loads, k_{mod} is given as 0.8.

It is also important to note that the behaviour of timber is not ductile and timber design is therefore different to steel and concrete design. The EC5[8] standard for designing timber structures is based on a simplified method of limit state, whereby characteristic values of load actions and material characteristics are adjusted by partial coefficients. Timber structures are analysed using elastic structural analysis techniques in ULS and SLS states. Thus, while the ULS loading is adopted as per EC5, the section modulus applied is the elastic, not the plastic, modulus.

As noted previously, for this rectangular section, the elastic modulus $bd^2/6$ – not the plastic modulus $bd^2/4$ – has been applied.

The depth of the timber plank d in mm is calculated as:

$$d = \sqrt{\frac{6 \times M \times \gamma_m}{f \times k_{mod}}} = \sqrt{\frac{6 \times 1.19 \times 10^3 \times 1.3}{18 \times 0.8}} = 25.39\text{mm}$$

Let us round this to 25mm, noting the arbitrary choice of a BM factor of 10 made earlier.

$$I_{(plank)} = \frac{bd^3}{12} = \frac{100 \times 2.5^3}{12} = 130.2\text{cm}^4/\text{m}$$

Ignoring vibration effects, timber deflection to reduce damage to brittle finishes is to be limited to $L/250$, otherwise $L/150$. The *Manual for the design of timber building structures to EC5*, however, notes that these deflection ratios are there to limit the curvature, rather than the absolute deflection[9]. As a continuous span of more than three bays, the span-to-deflection ratio for a UDL is to be reduced by 0.52:

$$\frac{L}{(0.52 \times 250)} = \frac{L}{130}$$

where:

$$\delta = \frac{1250}{130} = 9.62\,\text{mm}$$

Using the C value of 43.3 stated in Table 1:

$$\frac{43.3 \times 130}{200} = 28.15$$

$$I = CWL^3 = 28.15 \times 5.09 \times 1.25^3 = 280\text{cm}^4/\text{m}$$

A similar calculation for span/150 criteria yields I = 168cm^4/m.

Vibration performance in timber floors may be a concern[3] when spans are in excess of 4.0m. However, if a vibration check for residential premises is to be undertaken according to EC5, the maximum deflection to limit vibration for a 1.25m span should not exceed 1.8mm:

$$\frac{\text{span}}{\delta} = \frac{1250}{1.8} = 694.5$$

Table 1 notes that for a central point load:

$$C = 52 \times \frac{694.5}{200} = 180.75$$

where C = 52 and span/δ = 200.

As a continuous span of more than three bays, the span-to-deflection ratio for a central point load is to be reduced by 0.55:

$$I = 0.55 \times CWL^2 = 0.55 \times 180.75 \times 1 \times 1.25^2 = 155\,\text{cm}^4$$

To satisfy vibration criteria for residential premises, this I value varies between a strength of 130.2cm^4 and a deflection-to-span/150 ratio of 168cm^4 (Table 2).

Thus, for this short span, the deflection criteria are more onerous than the strength and vibration requirements.

They are further dependent on the type of damage to finishes tolerated. EC5 states that limiting deflections should be specified for each project and agreed upon with the client.

Design of secondary steel beam B1 with a 6.25m effective span

The beam loading is calculated as:

$$7.62\,\text{kN/m}^2 \times 1.25\,\text{m} = 9.50\,\text{kN/m} \qquad \text{(ULS)}$$
$$5.09\,\text{kN/m}^2 \times 1.25\,\text{m} = 6.36\,\text{kN/m} \qquad \text{(SLS)}$$

$$M_u = \frac{wl^2}{8} = \frac{9.5 \times 6.25^2}{8} = 46.4\text{kN/m}$$

Table 3: I values and relative beam depths for secondary steel beam

	Strength 275N/mm²	Deflection-to-span/360	Vibration-to-span/625
I (cm⁴)	1450	3892	6947
d (mm)	180	240	270
kg/m	21.9	30.7	42.3

Table 4: I values and relative beam depths for primary steel beam

	Strength 265N/mm²	Deflection-to-span/360	Vibration-to-span/750
I (cm⁴)	10 230	16 270	37 103
d (mm)	327	360	460.2
kg/m	43	57.1	82

For a fully restrained condition $Z_p = M/f$, where f = 275N/mm², for grade S275 steel (t<16mm):

$$Z_p = \frac{46.4 \times 10^3}{275} = 169\text{cm}^3$$

For an IPN 180 beam, Z_p = 187cm³.

Ignoring vibration effects, the deflection of the steel beam is to be limited to span/δ = 360.

$$I = CWL^3 = 2.23 \times 6.36 \times 6.25^3 = 3462\text{cm}^4$$

For an IPE O 240 beam, I = 3892cm⁴.

To limit vibration effects, the *Manual for the design of steelwork building structures to EC3*[5] recommends a total allowable deflection of 10mm for dance floors and gyms, with the natural frequency at least 5Hz. Thus:

$$\frac{\text{span}}{\delta} = \frac{6250}{10} = 625$$

$$C = 1.24 \times \frac{625}{200} = 3.88$$

where C = 1.24 for span/δ = 200 (Table 1).

$$I = CWL^3 = 3.88 \times 6.36 \times 6.25^3 = 6024\text{cm}^4$$

For an IPE O 270 beam, I = 6947cm⁴.

The natural frequency f_1 of a simply supported beam is derived from the basic equation:

$$f_1 = \frac{18}{\delta^{\frac{1}{2}}}$$

where δ is the maximum deflection due to the applied load and self-weight in mm. For IPE O 270:

$$\delta = 10\text{mm} \times \frac{6024}{6947} = 8.67\text{mm}$$

Therefore:

$$f_1 = \frac{18}{(8.67)^{\frac{1}{2}}} = 6.11\text{Hz}$$

The f_1 value of 6.11Hz confirms that this floor is adequate for use in a gym, as it is greater than the required minimum natural frequency of 5Hz.

Table 3 shows that if the structural design is to respect the vibration criteria, the amount of steel required is double that required solely to meet the strength criteria, resulting in a 100% increase in material costs. Similarly, meeting the deflection criteria would result in a 50% cost increase compared with the strength criteria alone.

Respecting the vibration criteria will create a stiffer grillage. As the steel joist is supported on the main steel girder, this increases the slope of the steel joist at its end, which in turn exacerbates the deflection of the secondary joist member. A prudent design would add half the deflection of the main beam to the calculated deflection of the joist when verifying that its deflection does not exceed the specified limits under a UDL[9].

Design of primary steel beam B2 with a 7.5m effective span

The edge loading is to consist of 12.5mm thick laminated glass, stiffened with glass fins at 1.25m centres, with a storey height of 3.5m and the self-weight of the beam:

$$0.0125\text{m} \times 25\text{kN/m}^3 \times 3.5\text{m} + 0.75\text{kN/m} = 1.84\text{kN/m}$$

The beam loading is calculated as:

$$\frac{9.50\text{ kN/m} \times \frac{6.25\text{m}}{2}}{1.25\text{m}} + 1.84\text{kN/m} \times 1.35 = 26.23\text{kN/m} \quad \text{(ULS)}$$

$$\frac{6.36\text{ kN/m} \times \frac{6.25\text{m}}{2}}{1.25\text{m}} + 1.84\text{kN/m} = 17.74\text{kN/m} \quad \text{(SLS)}$$

$$M_u = \frac{wl^2}{8} = 26.23 \times \frac{7.5^2}{8} = 184.43\text{kN/m}$$

A fully restrained condition is assumed, as the main beam is restrained by secondary beams at 1.25m centres: $Z_p = M/f$ where f = 265N/mm² for grade S275 steel (t>16mm):

$$Z_p = \frac{184.43 \times 10^3}{265} = 696\text{ cm}^3$$

For an IPE A 330 beam, Z_p = 702cm³.

Ignoring vibration effects, the deflection of the steel beam is to be limited to span/δ = 360 where C = 2.23 (Table 1):

$$I = CWL^3 = 2.23 \times 17.74 \times 7.5^3 = 16\,689\text{cm}^4$$

For an IPE O 360 beam, I = 16 270cm⁴.

The vibration effect in a gym can be limited by limiting the total deflection to within 10mm. Thus:

$$\frac{\text{span}}{\delta} = \frac{7500}{10} = 750$$

$$C = \frac{1.24 \times 750}{200} = 4.65$$

where C = 1.24 for span/δ = 200 (Table 1).

$$I = CWL^3 = 4.65 \times 17.74 \times 7.5^3 = 34\ 800\text{cm}^4$$

For a UK 457x191x82 universal beam, I = 37 103cm^4. In this case, the natural frequency f_1 is calculated by:

$$\delta = \frac{7500}{750} \times \left(\frac{34\ 800}{37\ 103}\right) = 9.38\text{ mm}$$

$$f_1 = \frac{18}{9.38} = 5.88\text{Hz}$$

The f_1 value of 5.88Hz confirms that the floor is suitable for use in a gym, as it is greater than the required minimum natural frequency of 5Hz.

These deflection calculations have been approximated to a UDL, not a series of point loads. Figure 2 in Part 1, from the BCSA[2], contains factors allowing the C value to be calculated for point loads.

To satisfy vibration criteria, while also reducing the risk of damaging the supported glazing panel, a deeper beam size is necessary than if designing to meet strength or deflection criteria. However, when designing for vibration criteria, a larger beam size is again required, resulting in a 90% increase in material costs compared to designing solely for strength criteria (Table 4).

For the 457x191x82 universal beam chosen, the actual span/δ is given by:

$$\frac{\text{span}}{\delta} = 750 \times \frac{34\ 800}{37\ 103} = 703$$

Rotation at the support is given by:

$$\frac{3.2}{703} = 0.00455\text{ rad}$$

For an internal environment, total rotation is given by:

$$0.00455\text{ rad} + 0.005\text{ rad (uncertainty factor)} = 0.00955\text{ rad}$$

This is considered a small rotation, as it lies outside the acceptable range of 0.015rad to 0.035rad, where 0.035rad is noted as 2°. Thus, this lightweight structure, in contrast to heavy loaded transfer slabs[10], generates a very reasonable rotation at the support.

On strength criteria, the bearing length of the steel beam is given by: reaction at support / (flange width × net bearing stress). For a flange width of 19.13 cm, on a dry mix mortar of 65% covering capacity and a grade 20/25 concrete padstone (with bearing stress given as 0.4 × 25N/mm^2), the bearing length is:

$$\frac{\left(\frac{1}{2} \times 7.5\text{m} \times 26.23\text{kN/m}\right)}{0.1913 \times (0.65 \times 0.4 \times 25\text{N/mm}^2)} = 79\text{ mm}$$

The total bearing length (as derived from BS 8110, Cl. 5.2.3[11]) is given by:

$$79\text{mm} + 25\text{mm (edge spalling)} + 7.5\text{m} \times 4\text{mm/m (beam tolerance)} = 134\text{mm}$$

This compares to a minimum specified bearing length of steel on concrete of 75mm.

The span-to-depth ratios in this design example have been utilised both to minimise damage to brittle materials (such as the extensive glazing) and to reduce the vibratory effects on users of the gym.

The final article in the series will present a preliminary design in composite floor construction for a three-storey warehouse, with insights into the deflection/vibration effects of bridge decks – a type of structure where the DL may be predominant. Hand calculations will be produced for the vertical sideways deflection of this three-storey warehouse.

References and further reading

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Erratum

In Part 1 of the series, published in the September 2014 issue of *The Structural Engineer*, Table 1 contained an error. The timber deflection coefficient C for a central point load at a span-to-deflection ratio of 1/500 was incorrectly given as 148.63; this should have read 130.2.

A corrected version of the article is available online at: www.thestructuralengineer.org

Material properties: effect on deflection, rotation and vibration – Part 3

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Introduction

This article is the third in a three-part series outlining the serviceability hand calculations to be undertaken in the preliminary structural design stage.

Structural design of a multi-storey, panelled, detached warehouse

Further to the structural design discussed in Part 2[1], this additional example refers to the serviceability requirements of a composite steel-concrete floor slab and the allowable horizontal drift of a medium-rise, three-storey warehouse with an effective span of 12m and a storey height of 5m (Figure 1).

A preliminary sizing calculation for the composite floor slab is undertaken according to EC4. The rectangular stress blocks for both the concrete slab and the steel section, which are obtained by multiplying the ultimate limit state (ULS) by the resultant lever arm, make for a simple calculation of the resisting moment. This hand calculation includes preliminary deflection and vibration checks. A series of technical articles on composite construction, published in *The Structural Engineer* in 2014[2–6], outlines the detailing required following the preliminary sizing analysis.

The panel structure also acts as a sway frame, with the overall side sway limited to $H/300$, where H is the overall height of the structure. The deflection of a vertical cantilever, as subjected to wind loading, creates a side sway deflection. The required panel thickness is dictated by this side sway deflection, together with the slenderness effects of the wall panels, which are exacerbated by the eccentricity of the floor slabs resulting from the fact that the slabs sit on the wall panels.

Here, the clear span is dictated by manoeuvring requirements; bracing elements may be absent due to deep-plan forms. The intermediate floor slabs are pin-jointed to the wall panels, while tying detail requirements may achieve a degree of stability. However, essentially, the floor slabs are props.

Composite-steel construction is regarded as one of the most economical systems for medium- to long-span construction, with a reduction in steel weight in the range of 30–50%[5]. It has benefited from the long experience in bridge construction.

For comparison, an alternative scheme using prestressed hollow-core panels across the 12m span is also described. In this alternative, the downstand of the steel beam is removed to create valuable storage height.

For the storage of heavy metal, assuming a storey height of 5m, the live load (LL) on the floor slabs is:

$$5\text{m} \times 4\text{kN/m}^2 = 20\text{kN/m}^2$$

The floor system adopted is a steel-composite construction with steel beams centred at 2.4m. For a total characteristic load of 33.89kN/m², the required thickness of the slab for the continuous spanning condition is 120mm. As the structural floor slab is to be used as the finished top surface in the warehouse, grade 30/37 concrete is specified, to be applied with a power float finish followed by hand trowelling.

For reinforced concrete, span-to-depth ratios are adopted instead of the span-to-deflection limits outlined here. For a span-to-depth ratio of 26, the 120mm concrete slab depth is the most economical solution, with the required reinforcement area based solely on strength criteria. A 100mm slab is adequate in deflection at a span-to-depth ratio of 26, but the area of steel required is 75% higher than for the 120mm slab;

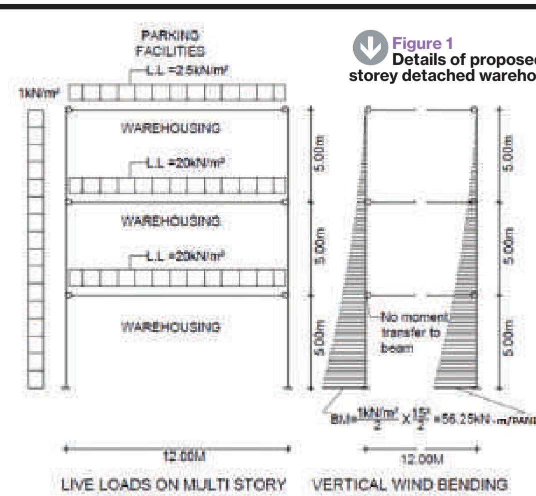


Figure 1 Details of proposed multi-storey detached warehouse

25% of this increase is to cater for deflection requirements.

In the case of reinforced concrete subjected to other physical properties, such as creep, span-to-depth ratios take precedence over the span-to-deflection ratio[7]. The span-to-depth ratio of 20 for simply supported spans is based on a span-to-deflection ratio of 1:250.

For prestressed slab panels, span-to-depth ratios vary from 30–40 for a span range of 6.0–13.0m. The high span-to-depth ratio refers to an office LL, with the low value referring to warehouse loading.

For composite construction, the serviceability requirements entail the calculation of the short-term, long-term and dynamic modular ratios. For preliminary sizing, the modular ratio is to be averaged out at 15 for imposed loads and 30 for dead loads (DLs). To calculate the natural frequency, the modular ratio is taken as 6, while 10% of the imposed load plus the DL is considered for the dynamic deflection.

Rackham *et al.*[8] note that, for internal beams in composite construction, the span-to-deflection ratios will be determined by the finishes. These are to be limited to span/360 for imposed loads and span/200 for total loads. For edge beams, the ratios are determined by the cladding. For glazing, these are to be limited to span/500. A minimum limit on natural frequency is proposed as 4 cycles/s for most building applications, except where there is vibrating machinery, and 3 cycles/s for car parks. The limit may be raised to 5 cycles/s for special buildings such as sport halls.

Hicks and Lawson[9] note that, for floors subjected to pedestrian traffic, the fundamental frequency is to be at least 3.55 cycles/s. This may be reduced to 3 cycles/s for steel-framed car parks.

Calculating the section of the composite steel beam

The floor load (which comprises the DL for a 120mm concrete slab and the storage load or LL of 20kN/m²) is calculated as:

DL	$0.12\text{m} \times 24\text{kN/m}^3$	$\times 1 = 2.88\text{kN/m}^2$	$\times 1.35 = 3.89\text{kN/m}^2$
LL	20kN/m^2	$\times 1 = \underline{20.0\text{kN/m}^2}$	$\times 1.5 \ = \underline{30.0\text{kN/m}^2}$
		(SLS) 22.88kN/m^2	(ULS) 33.89kN/m^2

where the ULS load is obtained by multiplying the serviceability limit state (SLS) load by partial safety factors defined in EC0.

The beam bending moment (BM) is calculated as:

$$(33.89\text{kN/m}^2 \times 2.4\text{m})\frac{12^2}{8} = 1464\text{kN/m}$$

The effective flange width b for this composite construction is given as span/4, but is limited to the centrelines between beams – in this case 2.4m is the limiting case.

As a trial section, adopt a grade S355 IPE 550 steel beam where: t_f = 17.2mm >16mm, f_y = 345N/mm².

The compressive resistance R_c of the slab is given by:

$$R_c = 0.85 f_{ck} \times b_{eff} \times h_c \times \frac{0.85 f_{ck}}{\gamma_c}$$

where:
f_{ck} is the characteristic cylinder strength of concrete = 30N/mm²
γ_c is the partial safety factor for concrete = 1.5

$$R_c = 0.85 \times \frac{30}{1.5} \times 2400 \times 120 \times 10^{-3} = 4896\text{kN}$$

The tensile resistance R_s of the steel section is given by:

$$R_s = A_a \times \frac{fy}{\gamma_s}$$

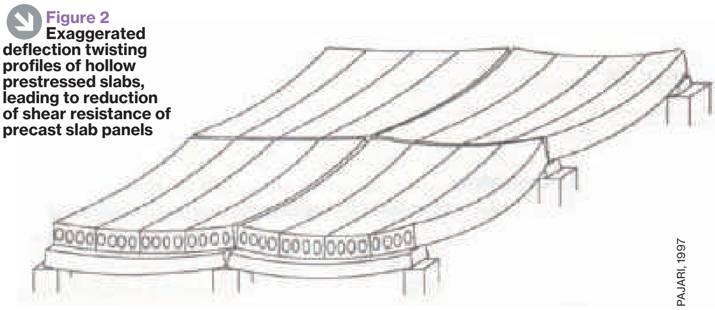


Figure 2 Exaggerated deflection twisting profiles of hollow prestressed slabs, leading to reduction of shear resistance of precast slab panels

where:
γ_s is the partial safety factor for concrete = 1.0

$$R_s = 134 \times \frac{345}{1} \times 10^{-1} = 4623\text{kN}$$

From the two respective stress blocks, it is noted that the plastic neutral axis falls within the base of the concrete slab.

The moment of resistance for this composite section is given by:

$$MR = 4623\text{kN} \times \frac{(0.55\text{m} + 0.12\text{m})}{2} = 1548\text{kN/m} > 1464\text{kN/m}$$

Alternative construction options to the solid slab are available. These include hollow-core or solid prestressed panels. If using prestressed panels, the whole depth of the concrete panel could be used for the concrete stress block; however, the effective width might be less than the 2.4m adopted (it is generally less than 1.5m)[5].

On the other hand, if profiled steel sheeting were adopted, then the overall depth of the overlying *in situ* concrete would not need to be used in the calculation of the concrete stress block, which could be reduced from 120mm to 70mm. The steel decking would also create a limitation on the placement of the steel shear studs, while there would be less surrounding concrete in the vicinity of the shear studs, resulting in a reduced shear capacity[3]. The SCI has a free composite beam checking tool available online[10], for the use of profiled steel sheeting as decking.

The deflection under total load – where span/200 and C = 1.24 (Table 1, Part 2) – is calculated as:

$$I = CWL^3 = 1.24 \times (22.88 \times 2.4) \times 12^3 = 117661\text{cm}^4$$

The deflection under an imposed load – where span/360 and C = 2.23 (Table 1, Part 2) – is calculated as:

$$I = CWL^3 = 2.23 \times (20.00 \times 2.4) \times 12^3 = 184965\text{cm}^4$$

For the deflection calculations, the modular ratio for total loading is averaged out at:

$$\frac{2.88 \times 30 + 20 \times 15}{22.88} = 16.88$$

The imposed load deflection, which is the limiting factor, is calculated as:

$$\frac{12000}{360} \times \frac{184965}{202906} = 30\text{mm}$$

(span/400 compared to required span/360)
where for a modular ratio of 15 SZS charts[11] provide:
I = 155 800cm⁴ < 184 965cm⁴ for an IPE 550 beam and 120mm concrete slab
I = 202 906cm⁴ > 184 965cm⁴ for an IPE 600 beam and 120mm concrete slab

The fundamental frequency may be calculated by first calculating the deflection given by the DL plus 10% of the imposed load:

$$W = (2.88 + 0.1 \text{ x } 20) \text{ x } 2.4\text{m} = 11.7\text{kN/m}$$

$$\frac{span}{200} = \frac{12000}{200} = 60\text{mm}$$

$$I = CWL^3 = 1.24 \times 11.7 \times 12^3 = 25069\text{cm}^4$$

The actual deflection is:

$$60\text{mm} \times \frac{25069}{250300} = 6\text{mm}$$

where for a modular ratio of 6 SZS charts[11] provide:
I = 250 300cm^4 > 184 965cm^4 for an IPE 600 beam and 120mm concrete slab

The fundamental frequency f_t that this floor is subjected to is calculated by:

$$f_t = \frac{18}{6^{\frac{1}{2}}} = 7.35\text{Hz} > 3\text{Hz}$$

Thus, the vibration criteria are also satisfied. However, to satisfy the strength and deflection criteria, a 15% increase in material costs has to be incurred, due to an increase in section size from IPE 550 to IPE 600.

In addition, to satisfy the strength, deflection and vibration criteria for this warehouse loading scenario, the span-to-depth ratio works out as:

$$\frac{12000}{(600 + 120)} = 16.67$$

Once the preliminary sizing has been ascertained, the final design must be produced to conform to EC4.

Bridge decks

In composite construction of bridge decks, span-to-deflection ratios higher than 1:200 are advisable, although US and European recommendations differ[12]. US recommendations are for the LL span-to-deflection ratio not to exceed 1:800 for a vehicular bridge, or 1:1000 for a pedestrian bridge. On the other hand, no ratio restrictions exist in European bridge specifications. It is suggested in the literature that the higher limit for pedestrian bridges was inspired by a baby being woken on a bridge by the bridge vibration.

The fundamental frequency of a vehicular bridge is quoted in the range of 3Hz. The natural frequency of the bridge should be outside the range of 0.5–1.5 times the fundamental frequency, to avoid intolerable dynamic conditions due to resonance.

US limits on allowable LL deflection appeared in the early 1930s when a study attempted to link the unpleasant vibrations felt on a sample of bridges built during that era. The study concluded that structures with vibrations deemed unacceptable by a subjective human response had deflections that exceeded L/800. Human reactions to vibrations are classified as either physiological or psychological. Psychological discomfort results from unexpected motion, but physiological discomfort, such as seasickness, results from a low-frequency, high-amplitude vibration. The limits described here produce beam-to-depth ratios varying from L/15 to L/20.

These recommendations are consistent with the warehouse flooring system adopted in this example, where the span-to-depth ratio is 16.67 and the natural frequency of 7.35Hz lies outside the range of 0.5–1.5 times the fundamental frequency, so resonance does not occur.

Furthermore, the deflection limit of 1:800 allows for a rotation in radians (Equation 3, Part 1) of:

$$\frac{3.2}{800} = 0.004\text{rad}$$

where 3.2 is the constant for a uniform distributed load (UDL), decreasing to 3 for a point load.

The total rotation at the bearing may be obtained by noting that in bridge decks the LL:DL ratio lies between approx. 0.5 (for medium spans) and 1.0 (for short spans). In addition, an allowance for thermal camber at 0.0015rad, together with a value for uncertainty taken at 0.01rad, should be allowed made[13]. In the simple support condition, this gives a total rotation in radians at the bearing of:

$$0.01 \text{ (uncertainty)} + 0.0015 \text{ (thermal camber)} + 2 \times 0.004 \text{ (traffic)} = 0.02\text{rad}$$

As noted in Part 2, this is not to be considered a small rotation although it lies within the acceptable range 0.015–0.035rad.

Alternative design proposal using hollow-core prestressed slab on a rigid support

For the 12m effective span, a hollow-core prestressed slab with a depth of 525mm, acting compositely with a 100mm grade 30/37 concrete topping, provides a safe load of 23.9kN/m^2 and a shear capacity of 226.5kN/m. This exceeds the superimposed serviceable load of 22.88kN/m^2, thereby satisfying the required strength criteria.

The upward camber with DL only applied is 14.32mm (δ/span = 1/838).

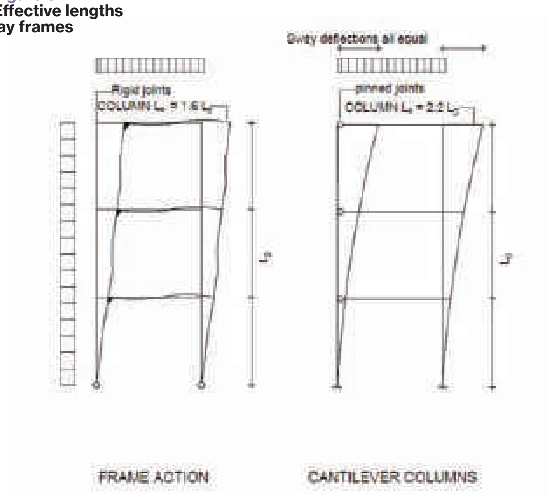
The resulting downward deflection on application of the full load is 23.07mm (δ/span = 1/522).

The span-to-depth ratio for this scheme is 12 000/525 = 22.85.

High deflection ratios are important in precast, prestressed hollow slabs supported on non-rigid supports. Tests show the reduction in shear resistance to these precast slabs to be in the region of 40–77%[14]. This is due to the transverse deformation of the slab ends resulting from the deflection of the supporting beam (Figure 2)[15]. Deflection-to-span ratios exceeding 1:1000 are an advantage in these situations.

This alternative proposal is therefore a workable scheme, which satisfies both the strength and serviceability criteria. In choosing between the two schemes, the high DL imposed by the prestressed solution should be taken into account. The DL for these panels with a 12m span is 50% of the LL, which is comparable to bridge construction. For the composite construction scheme, the DL only approximates to 14.5% of

Figure 3
Effective lengths for sway frames



the LL. The prestressed slab scheme produces an increase in storage height of 95mm (overall thickness of 525mm + 100mm) compared to the composite-steel scheme (600mm + 120mm), but this is nominal for an overall storey height of 5m. The costs of the schemes, which will be affected by fire protection required, will be decisive in determining between them.

The lateral stability of this multi-storey warehouse is affected by the overall effective length of the wall panels. The overall effective length for this sway structure taken at $2.2L_o$, where L_o is the storey height (Figure 3)[16]. A reduction factor ϕ for the vertical load-bearing capacity is obtained from the masonry EC6. This reduction factor ϕ is dependent on both the slenderness ratio and the eccentricity at the top of the wall. These vertical load calculations are well catered for in *The calculation of eccentricities in load bearing walls*[17]. The authors now present a hand calculation to limit the amount of horizontal drift.

For a preliminary hand calculation to be performed, the required I value at the base of the wall panel is again given by: $I = CwL^3$

The C value for a steel cantilever now works out at 18.3 for a UDL and 48.3 for an edge point load. Table 1 presents C values for calculating the cantilever horizontal drift for members in steelwork with a Young's modulus E value of 210N/mm², in timber with an E value of 8kN/mm², and in concrete with an E value of 30kN/mm². Some types of natural masonry, such as sandstone or limestone, could have Young's modulus E values similar to concrete, so the calculation that follows may also apply to certain natural masonry units.

As an example, let us consider the deflection δ of a cantilever steel beam (E = 210kN/mm²) and a reinforced concrete beam (E = 30kN/mm²) to be limited to span/300. For a cantilever beam of effective span L, subjected to a UDL of w, the top deflection δ is given by:

$$\delta = \frac{L}{300} = \frac{wL^4}{8EI}$$

Assuming consistent units throughout in mm and then converting moment of inertia I to cm⁴ by dividing by 10⁴:

$$I = \frac{300 \times w \times L^3 \times 1000^3}{(8 \times E \times 1000) \times 10^4} = CwL^3$$

where:
w is in kN/m
L is in m

$$C = \frac{300 \times 10^9}{(8 \times 210 \times 10^7)} = 17.86$$

The C value of 17.86 conforms to the value quoted in Table 1.

For a concrete cantilever section, to restrict the deflection to span/300:

$$C = \frac{17.86 \times 210}{30} = 125$$

The C value of 125 conforms to the value quoted in Table 1.

15m high panel structure subjected to wind load of 1kN/m²

The side sway is to be limited to:

$$\frac{15\text{m}}{300} = 50\text{mm}$$

If this wind pressure is to be distributed onto the two supporting concrete panel walls, the moment of inertia I required is given by:

$$I = \frac{CWL^3}{2}$$

$$I = \frac{125 \times 1\text{kN/m}^2 \times 15^3}{2} = 210937\text{cm}^4/\text{m}$$

where C =125 (Table 1).

To satisfy this I value, the thickness h of the concrete wall panel for serviceability requirements is estimated as:

$$h = \sqrt[3]{\frac{12 \times 210937}{100}} = 29.36\text{cm} \ \ (\text{or } 30\text{cm})$$

where:

$$I = \frac{bh^3}{12}$$

A vertical load analysis will then have to be undertaken to include the direct dead and imposed loads and the wind-induced BM, all catering for the eccentricity induced at the lowest level. The eccentric load combination, compounded by the slenderness ratio, will then be subjected to a reduction factor ϕ. The slenderness ratio is given as:

$$\frac{L_e}{h} = \frac{2.2L_o}{h} = 2.2 \times \frac{5\text{m}}{0.3\text{m}} = 36.67$$

EC6 gives the maximum allowable slenderness ratio as 30, with a reduction factor ϕ_m at the centre of the panel storey height varying from 0 for an eccentricity of $0.4h$ up to a value of 0.38 for a nominal eccentricity of $0.05h$.

Thus, to satisfy slenderness effects, the minimum wall panel thickness is given as:

$$\frac{2.2 \times 5\text{m}}{30} = 0.367\text{m}$$

This is greater than the 0.3m noted earlier for sway limitations.

Strength calculations should then follow to verify the load-bearing capacity of the wall panel.

According to the rule of thumb for the natural frequency of framed vertical structures described in Part 2, this three-storey structure appears to have a natural frequency in the range of 2.5Hz. If compared to vibrating table tests performed on various masonry buildings, adobe buildings are quoted as having a forcing frequency of 6Hz, with a six-storey masonry building having a fundamental frequency of 2Hz, increasing to 5.5Hz for a two-storey masonry building.

Table 1: Updated deflection coefficient C for calculating moment of inertia for a cantilever span condition

Span-to-deflection ratio	Steel E = 210kN/mm²	Concrete E = 30kN/mm²	Timber E = 8kN/mm²
1/300 (UDL)	17.86	125	469
1/300 (point load)	47.15	330	1238

Conclusions and recommendations

The method described in Parts 1–3 revolves around Equation 1 in Part 1[18], which calculates the moment of inertia I necessary for deflection criteria to be abided by. This is undertaken in a structural engineer's parlance, with the units given in m and kN to give I in cm⁴. The method produces coefficients, meaning that it is not restricted to steelwork but can also be applied to other materials. For concrete, this method is applicable only to establish the requirements of horizontal side sway, as vertical deflections are based on span-to-depth ratios not deflection-to-span ratios.

Parts 1–3 move from strength calculations to the SLS, while covering various structural materials by adopting a universal formula with a varying constant, which depends on the material. The preliminary design outlined with 'back-of-the-envelope calculations' also helps the structural designer to perform further dynamic calculations in addition to the static calculations which are usually performed in the design office. The preliminary design stage now allows the rotations and vibrations imposed on the structure to be quantified, as it is an easy step to move onto rotation and vibration effects once a deflection-to-span ratio has been determined. At this stage, the designer and client can discuss the serviceability requirements gauged to be important for the final project.

The article then examines interactions between structural materials to limit unsightly secondary failures. The outline of the characteristics of the various structural groups of materials, which is presented in Table 2, Part 1, helps to establish the values of the important design properties. This should make it easier for the design engineer to move from one structural material to another. Knowledge of material properties will help the structural designer to choose the appropriate structural material, based on efficiency criteria and sustainability properties, rather than simply using the material they feel most comfortable with.

The hand calculations presented in Equations 1–5 in Part 1, in conjunction with the BCSA deflection coefficients chart[17] (Figure 2, Part 1) should help practices to design constructions that will not suffer unwarranted cracking or unwelcome movement when in use, thus avoiding lengthy litigation over serviceability failure. As outlined briefly in the introduction to Part 1, crack width may not be the defining factor, although this depends on the length, shape and density of cracks. Cracks may have a negative aesthetic impact or they may need to be filled to reduce penetration of sound and odours or the passage of fire. People are sensitive to distinctly perceptible vibrations in an office or residential environment, but will accept vibrations approximately 10 times greater in an active environment, such as when dining beside a dance floor or lifting weights in a gym. Finally, the age of the building also comes into the equation. The older the building, the less sensitive its users are to existing cracks, deflections and vibration effects.

On the other hand, in bridge works, no evidence of serious structural damage is attributable to excessive LL deflection. Human psychological reaction to vibration and deflection is a more significant issue than that of structural durability.

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Erratum

In Part 2 of the series, published in the October 2014 issue of *The Structural Engineer*, the section on the design of the primary steel beam contained an error. The edge loading calculation incorrectly gave the laminated glass thickness as 0.125m; this should have read 0.0125m. Correcting this error would allow a 457×191×82 universal beam to be chosen.

A corrected version of the article is available online at: www.thestructuralengineer.org