

# REINFORCED CONCRETE & BASIC RESTRAINED STEEL BEAM DESIGN

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**‘Course B’ Module 1 -  
Introduction to the material  
properties, durability,  
resistance to fire, minimum  
cover, nominal areas and bar  
spacing as outlined in EC2**

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12<sup>th</sup> /14<sup>th</sup>  
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Table 1: Partial factors for materials for ultimate limit states

Design situations	$\gamma_c$ for concrete	$\gamma_s$ for reinforcing steel	$\gamma_s$ for prestressing steel
Persistent & Transient	1,5	1,15	1,15
Accidental	1,2	1,0	1,0

# Table 2 Strength and deformation characteristics

Strength classes for concrete															Analytical relation / Explanation
$f_{ck}$ (MPa)	12	16	20	25	30	35	40	45	50	55	60	70	80	90	
$f_{ck,cube}$ (MPa)	15	20	25	30	37	45	50	56	60	67	75	85	96	105	2.8
$f_{cm}$ (MPa)	20	24	28	33	38	43	48	53	58	63	68	78	88	98	$f_{cm} = f_{ck} + 8$ (MPa)
$f_{ctm}$ (MPa)	1,6	1,9	2,2	2,6	2,9	3,2	3,5	3,8	4,1	4,2	4,4	4,6	4,8	5,0	$f_{ctm} = 0,30 \times f_{ck}^{(2/3)} \leq C50/60$ $f_{ctm} = 2,12 \cdot \ln(1 + (f_{cm}/10)) > C50/60$
$f_{ctk,0,05}$ (MPa)	1,1	1,3	1,5	1,8	2,0	2,2	2,5	2,7	2,9	3,0	3,1	3,2	3,4	3,5	$f_{ctk,0,05} = 0,7 \times f_{ctm}$ 5% fractile
$f_{ctk,0,95}$ (MPa)	2,0	2,5	2,9	3,3	3,8	4,2	4,6	4,9	5,3	5,5	5,7	6,0	6,3	6,6	$f_{ctk,0,95} = 1,3 \times f_{ctm}$ 95% fractile
$E_{cm}$ (GPa)	27	29	30	31	33	34	35	36	37	38	39	41	42	44	$E_{cm} = 22[(f_{cm}/10)]^{0,3}$ ( $f_{cm}$ in MPa)
$\epsilon_{ct}$ (‰)	1,8	1,9	2,0	2,1	2,2	2,25	2,3	2,4	2,45	2,5	2,6	2,7	2,8	2,8	see Figure 3.2 $\epsilon_{ct}^{(f_{cm})} = 0,7 f_{cm}^{-0,21} \leq 2,8$ (‰)
$\epsilon_{cu1}$ (‰)	3,5									3,2	3,0	2,8	2,8	2,8	see Figure 3.2 for $f_{ck} \geq 50$ Mpa $\epsilon_{cu1}^{(f_{cm})} = 2,8 + 27[(98 - f_{cm})/100]^4$
$\epsilon_{cu2}$ (‰)	2,0									2,2	2,3	2,4	2,5	2,6	see Figure 3.3 for $f_{ck} \geq 50$ Mpa $\epsilon_{cu2}^{(f_{cm})} = 2,0 + 0,085(f_{cm} - 50)^{0,22}$
$\epsilon_{cu2}$ (‰)	3,5									3,1	2,9	2,7	2,6	2,6	see Figure 3.3 for $f_{ck} \geq 50$ Mpa $\epsilon_{cu2}^{(f_{cm})} = 2,6 + 35[(90 - f_{cm})/100]^4$
$n$	2,0									1,75	1,6	1,45	1,4	1,4	for $f_{ck} \geq 50$ Mpa $n = 1,4 + 23,4[(90 - f_{cm})/100]^4$
$\epsilon_{cu3}$ (‰)	1,75									1,8	1,9	2,0	2,2	2,3	see Figure 3.4 for $f_{ck} \geq 50$ Mpa $\epsilon_{cu3}^{(f_{cm})} = 1,75 + 0,55[(f_{cm} - 50)/40]$
$\epsilon_{cu5}$ (‰)	3,5									3,1	2,9	2,7	2,6	2,6	see Figure 3.4 for $f_{ck} \geq 50$ Mpa $\epsilon_{cu5}^{(f_{cm})} = 2,6 + 35[(90 - f_{cm})/100]^4$

Table 3. Characteristic compressive strength,  $f_{ck}$ , mean compressive strength,  $f_{cm}$ , characteristic tensile strength,  $f_{ctk}$ , (in MPa) and secant modulus of elasticity,  $E_{cm}$ , (in MN/m<sup>2</sup>) of concrete, by strength class.

Strength class of concrete	C20/25	C30/37	C40/50	C50/60	C60/75	C70/85	C80/95	C90/105
$f_{ck}$	20.00	30.00	40.00	50.00	60.00	70.00	80.00	90.00
$f_{cm}$	28.00	38.00	48.00	58.00	68.00	78.00	95.00	98.00
$f_{ctk0.05}$	1,5	2,0	2,5	2,9	3,1	3,2	3,4	3,5
$f_{ctk0.95}$	2,9	3,8	4,6	5,3	5,7	6,0	6,3	6,6
$E_{cm}$	30.00	33.00	35.00	37.00	39.00	41.00	42.00	44.00

The design values of the compressive and tensile strength of concrete are calculated from the following expressions

$$f_{cd} = \alpha_{cc} f_{ck} / \gamma_c, \quad \text{and} \quad f_{ctd} = \alpha_{ct} f_{ctk0.05} / \gamma_c$$

where:

$\gamma_c$  is the partial safety factor for concrete. The recommended value is 1,5 for persistent and transient situations and 1,2 for accidental situations.

$\alpha_{cc}$ ,  $\alpha_{ct}$  are coefficients taking account of long-term effects and the way the load is

# TIME-DEPENDENCE OF CONCRETE MECHANICAL PARAMETERS – COMPRESSIVE STRENGTH

When concrete compressive strength is needed for ages other than 28 days (e.g.: demoulding, prestress) and no tests are to be conducted, the characteristic value at time  $t$ ,  $f_{ck}(t)$ , may be approximated from the expression:

$$f_{ck(t)} = f_{cm(t)} - 8 \text{ [kN/m}^2\text{]}, \quad \text{for } 3 < t < 28 \text{ days,}$$
$$f_{ck(t)} = f_{ck}, \quad \text{for } t \geq 28 \text{ days.}$$

Compressive strength depends on the kind of cement and the curing temperature and conditions. When the concrete is cured under normal conditions, the mean value may be estimated as:

$$f_{cm(t)} = \beta_{cc}(t) f_{cm}$$

Taking:

$$\beta_{cc}(t) = \exp\{s[1-(28/t)^{1/2}]\};$$

where:

$f_{cm(t)}$  is the mean concrete compressive strength at age  $t$  (in days),

$f_{cm}$  is the mean concrete compressive strength at 28 days,

$\beta_{cc}(t)$  is a coefficient depending of the age and type of cement,

$t$  is the age in days,

$s$  is a coefficient depending on the cement class

$s = 0,2$  for cement classes: CEM 42.5 R, CEM 53.5N and CEM 53.5R (Class R)

$s = 0,35$  for classes CEM 32.5 R. CEM 42.5 N (Class N)

$s = 0,38$  for classes CEM 32.5 N (Class S)

# Table 4 - Characteristic concrete compressive strength at different ages (in kN/m<sup>2</sup>).

	$f_{ck}$ [kN/m <sup>2</sup> ]	Age [days]		
		3	7	14
Class R. $s = 0,20$	30	17,2	23,1	27,0
	50	30,5	39,5	45,4
	70	43,7	55,9	63,8
Class N. $s = 0,35$	30	10,5	18,8	24,9
	50	20,3	32,9	42,2
	70	30,0	47,0	59,5
Class S. $s = 0,38$	30	9,4	18,0	24,5
	50	18,6	31,7	41,6
	70	27,7	45,3	58,6

# TIME-DEPENDANCE OF CONCRETE MECHANICAL PARAMETERS – TENSILE STRENGTH

The development of tensile strength with time is strongly influenced by curing and drying conditions as well as by the dimensions of the structural members. As a first approximation it may be assumed that the tensile strength  $f_{ctm(t)}$  is equal to:

$$f_{ctm(t)} = (\beta_{cc}(t))\alpha \cdot f_{ctm} \quad (3.4)$$

where  $\beta_{cc}(t)$  follows from Expression (3.2) and

$$\alpha = 1 \text{ for } t < 28$$

$$\alpha = 2/3 \text{ for } t \geq 28. \text{ The values for } f_{ctm} \text{ are given in Table 3.1.}$$

**Note:** Where the development of the tensile strength with time is important it is recommended that tests are carried out taking into account the exposure conditions and the dimensions of the structural member.

# TIME-DEPENDANCE OF CONCRETE MECHANICAL PARAMETERS – ELASTIC MODULUS

The value of the mean elastic modulus of concrete at 28 days,  $E_{cm}$ , can be estimated from mean compressive strength with the following expression:

$$E_{cm} [MN/m^2] = 22 [(f_{cm}/10)]^{0,3} ; f_{cm} \text{ in } [kN/m^2].$$

The following equation can be used to compute the variation of the elastic modulus with time:

$$E_{cm}(t) = (f_{cm}(t)/f_{cm})^{0,3} E_{cm}$$



# Table 5 Mean elastic modulus of concrete at different ages (in MN/m<sup>2</sup>)

	$f_{ck}$ [MPa]	Age [days]			
		3	7	14	28
Class R. $s = 0,2$	30	29,0	30,9	32,0	32,8
	50	33,0	35,1	36,4	37,3
	70	36,0	38,4	39,7	40,7
Class N. $s = 0,35$	30	26,5	29,6	31,4	32,8
	50	30,0	33,6	35,7	37,3
	70	32,8	36,7	39,0	40,7
Class S. $s = 0,38$	30	26,0	29,3	31,3	32,8
	50	29,5	33,3	35,6	37,3
	70	32,2	36,4	38,9	40,7

# TABLE 6 – EXPOSURE CONDITIONS

Exposure class	Conditions	Examples
<b>Exposed to air and moisture</b>		
XC1	Dry or permanently wet	Interior of buildings for normal habitation (e.g. homes, offices)
XC2	Wet, rarely dry	Completely buried in non-aggressive soil
XC3/4	Moderate humidity, or cyclic wet/dry	Normal outdoor condition, indoor subject to high humidity
<b>In contact with water containing chlorides, including de-icing salt</b>		
XD1	Moderate humidity	Highway structures away from direct spray
XD2	Wet, rarely dry	Totally immersed in water containing chlorides
XD3	Cyclic wet/dry	Highway structures within 10m of carriageway, including car parks
<b>In contact with sea water or air carrying salts from sea water</b>		
XS1	Exposed to airborne salt but not in direct contact with seawater	Structures in coastal areas
XS2	Permanently submerged	Below mid-tide level
XS3	Tidal, splash and spray zones	Upper tidal, splash and spray zones

# TABLE 7 – NOMINAL CONCRETE COVER TO REINFORCEMENT FOR CONCRETE MADE WITH OPC FOR A 50-YEAR DESIGN LIFE

Exposure class	Nominal cover to all reinforcement (mm)								
	25	30	35	40	45	50	55	60	
XC1	C20/25 0.7 240								
XC2			C25/30 0.65 260						
XC3/4		C40/50 0.45 340	C32/40 0.55 300	C28/35 0.6 280	C25/30 0.65 260				
XD1			C40/50 0.45 360	C32/40 0.55 320	C28/35 0.60 300				
XD2			C40/50 0.40 380	C32/40 0.50 340	C28/35 0.55 320				
XD3						C45/55 0.35 380	C40/50 0.40 380	C35/45 0.45 260	
XS1				C50/60 0.35 380	C40/50 0.45 360	C35/45 0.50 340			
XS2				C40/50 0.40 380	C32/40 0.50 340	C28/35 0.55 320			
XS3							C45/55 0.35 380	C40/50 0.40 380	
Key to entries:		C40/50 0.40 380	— Minimum concrete grade — Maximum water/cement ratio — Minimum cement content in kg/m <sup>3</sup>						

Source: Table NA3 of UK National Annex to EC2 Part 1-1 with  $\Delta_{cl,pc} = 10$  mm.

## Notes:

- (1) The specified cover should be provided to all reinforcement.
- (2) When the work is carried out by a company with a recognised quality control system, the nominal cover values may be reduced by 5mm.

# Table 8 – Resistance to fire: minimum sizes and minimum axis distances for columns and for simply supported slabs

Element		R30	R60	R90	R120	R180	R240
Column exposed to fire on all sides	Minimum cross-section dimension (mm)	200	250	350	350	450	–
	Minimum axis distance (mm)	32	46	53	57	70	–
Simply supported slab with plain soffit	Minimum thickness (mm)	60	80	100	120	150	175
	Minimum axis distance (mm)	10*	20	30	40	55	65

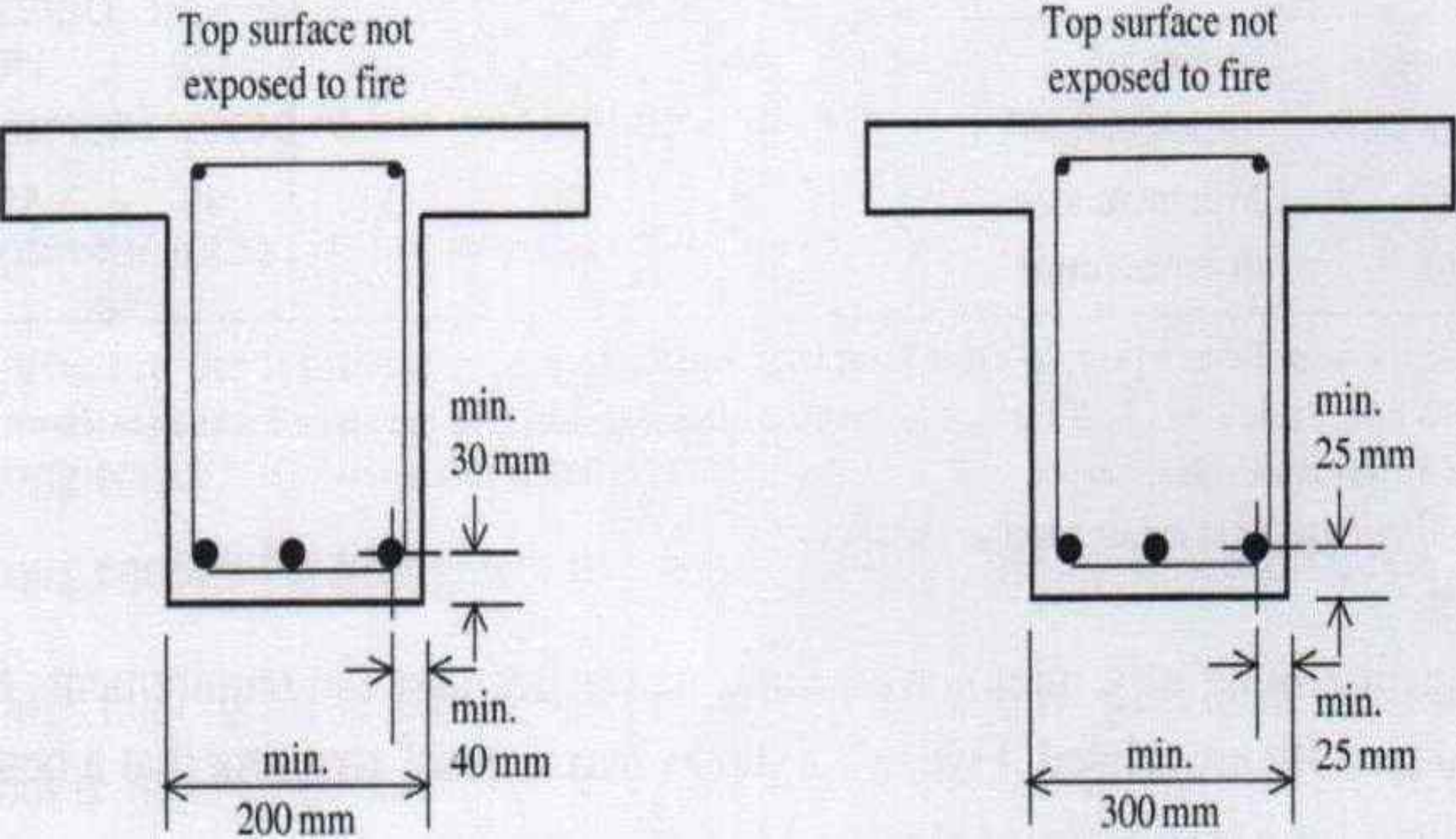


# Table 9 – Resistance to Fire: minimum sizes and minimum axis distances for simply supported beams exposed to fire on three faces

Standard fire resistance	Parameter	Possible combinations (mm)			
R30	Minimum breadth	80	120	160	200
	Minimum axis distance from soffit	25	20	15*	15*
	Minimum axis distance from side	35	30	25	15*
R60	Minimum breadth	120	160	200	300
	Minimum axis distance from soffit	40	35	30	25
	Minimum axis distance from side	50	45	40	25
R90	Minimum breadth	150	200	300	400
	Minimum axis distance from soffit	55	45	40	35
	Minimum axis distance from side	65	55	50	35
R120	Minimum breadth	200	240	300	500
	Minimum axis distance from soffit	65	60	55	50
	Minimum axis distance from side	75	70	65	50
R180	Minimum breadth	240	300	400	600
	Minimum axis distance from soffit	80	70	65	60
	Minimum axis distance from side	90	80	75	60
R240	Minimum breadth	280	350	500	700
	Minimum axis distance from soffit	90	80	75	70
	Minimum axis distance from side	100	90	85	70

When the axis distance is 70 mm or more, surface reinforcement of 4 mm bars at 100 mm spacing should be provided to prevent the surface concrete falling off during the fire.

# Figure 1 – Two ways of ensuring that has R60 fire resistance



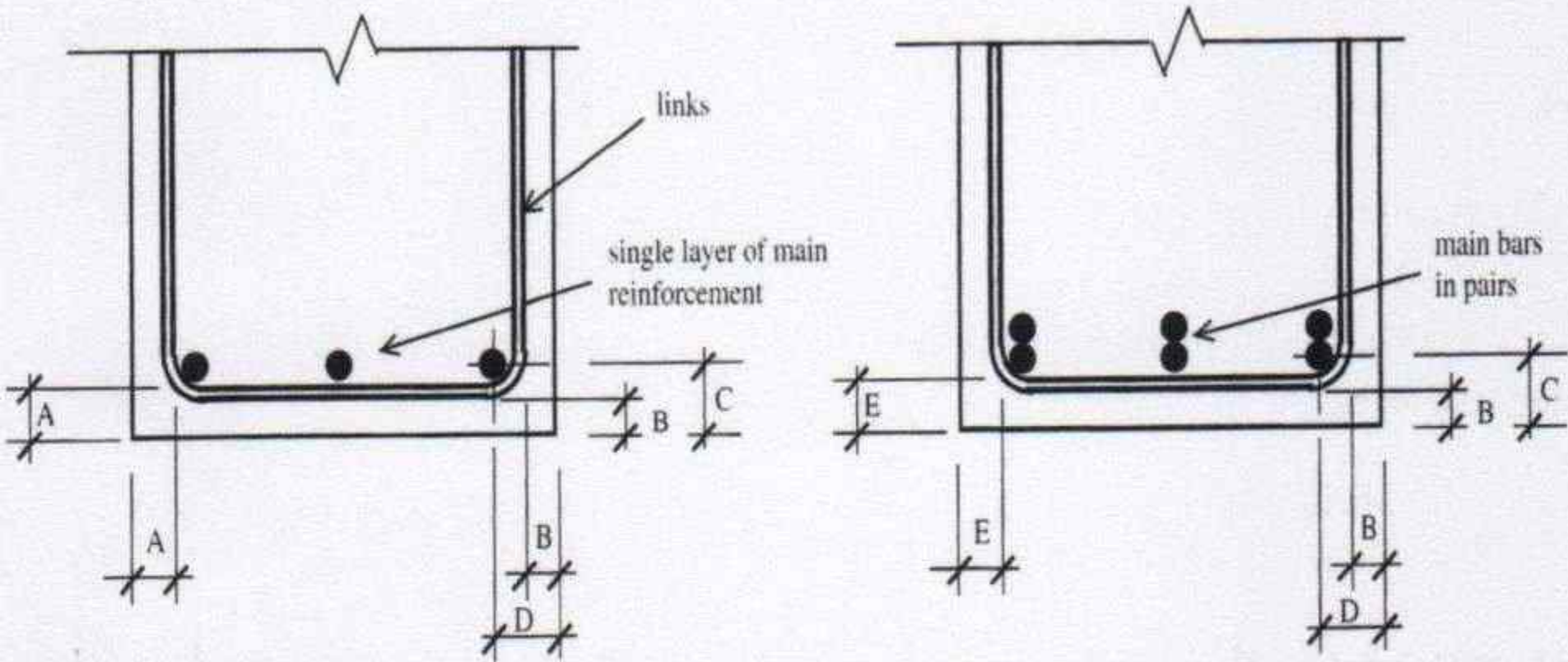
# Table 10 – Factors determining minimum cover to reinforcing bars

Placing of concrete	Not less than the diameter of the bar $\phi + 10$ mm
Bond between steel and concrete	Not less than the diameter of the bar $\phi + 10$ mm If the bars are bundled, the minimum cover should be: <ul style="list-style-type: none"><li>• <math>1.4\phi + 10</math> mm for a 2-bar bundle</li><li>• <math>1.7\phi + 10</math> mm for a 3-bar bundle</li><li>• <math>2.0\phi + 10</math> mm for a 4-bar bundle</li></ul>
Durability	See exposure classes in Table 3.9 and requirements in Table 3.10
Fire resistance	See Table 3.11 for slabs and columns, Table 3.12 for beams

Note that the first three of these factors determine the minimum cover to all bars while fire resistance requirements are given in terms of axis distance to the main reinforcement.



# Figure 2 – Factors determining cover to reinforcement



- A = cover to a single main bar, not less than bar diameter + 10 mm. Determined by requirement for concrete compaction and for bond.
- B = cover to all reinforcement, determined by durability requirements.
- C, D = axis distance to main bar, determined by fire resistance requirement.
- E = cover to group of 2 bars, not less than 1.4 bar diameters + 10 mm. Determined by requirement for concrete compaction and for bond.



# Reinforcement Requirements - 1

**Table 11:** Minimum percentage of tensile reinforcement in beams and slabs

Conc. strength $f_{ck}$ (N/mm <sup>2</sup> )	25	28	30	32	35	40	45	50
Minimum % of reinforcement	0.14	0.15	0.15	0.16	0.17	0.19	0.20	0.22

This table uses  $0.016f_{ck}^{2/3}$  as recommended by the *IStructE manual for the design of concrete building structures to Eurocode 2* in place of  $0.0156f_{ck}^{2/3}$  in EC2.

**Table 12:** Maximum bar size or maximum bar spacing for 0.3-mm crack width limit for load-induced cracking in beams and in slabs more than 200 mm thick

Steel stress (N/mm <sup>2</sup> ): see note 1	160	200	240	280	320	360	400
Max. bar size	H32	H25	H16	H12	H10	H8	H6
Max. bar spacing (mm)	300	250	200	150	100	50	–

Notes:

- (1) These rules do not apply to secondary or distribution reinforcement.
- (2) The steel stress can be taken as  $435(G_k + 0.8Q_k)/(1.35G_k + 1.50Q_k)$  N/mm<sup>2</sup>, or conservatively as 320 N/mm<sup>2</sup>.
- (3) Cracks may be controlled by meeting either the max. bar spacing requirement *or* the max. bar size requirement. It is not necessary to meet both requirements. For example, if the steel stress is 280 N/mm<sup>2</sup> then either bars of size H12 or smaller can be used at any spacing or bars of any size can be used at a spacing of 150 mm or less.
- (4) Data are from Tables 7.2N and 7.3N of EC2 Part 1-1.

# Reinforcement Requirements - 2

**Table 13: Maximum bar spacing: other provisions**

Location	Maximum bar spacing
Reinforcement in slabs 200 mm thick or less	Main reinforcement, $3d$ but not more than 400 mm Secondary reinforcement, $3.5d$ but not more than 450 mm
Additional side reinforcement in beams more than 1000 mm deep	See EC2

**Table 14: Maximum amount of reinforcement**

Location	Maximum amount of reinforcement
Slab or beam, tension reinforcement	4% other than at laps
Slab or beam, compression reinforcement	4% other than at laps
Column	4%, or 8% at laps

**Table 15: Minimum clear space between bars**

Gap between bars (except at laps) should be at least:	<ul style="list-style-type: none"><li>• the maximum bar size <math>\phi</math></li><li>• 20 mm</li><li>• aggregate size + 5 mm (=25 mm when using the normal aggregate size of 20 mm)</li></ul>
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# Maximum & Minimum Reinforcement in Columns

**Table 16: Limits on longitudinal reinforcement in columns**

Area of longitudinal reinforcement.	Not less than or Not more than or	$0.12N/f_{yk}$ $0.002bh$ $0.04bh$ generally $0.08bh$ at laps
Size of longitudinal reinforcement	Not less than	12 mm
Number of longitudinal bars	Not fewer than	4 in a square column 6 in a circular column

**Table 17: Limits on ties in reinforced concrete columns**

Size of ties	Not less than or	main bar size/4 6 mm
Spacing of ties generally	Not less than or or	$20 \times$ main bar diameter the least column dimension 400 mm
Spacing of ties over a height equal to the larger dimension of the column above or below a beam or slab	Not more than or or	$12 \times$ main bar diameter $0.6 \times$ the least column dimension 240 mm
Spacing of ties where longitudinal bars exceeding 12 mm in size are lapped	Not more than or or	$12 \times$ main bar diameter $0.6 \times$ the least column dimension 240 mm

# Properties of steel reinforcing bars

**Table 18: Cross-section areas, maximum lateral dimension and mass of reinforcing bars**

Bar designation	H6	H8	H10	H12	H16	H20	H25	H32	H40
Cross-section area (mm <sup>2</sup> )	28	50	79	113	201	314	491	804	1257
Maximum lateral dimension (mm)	8	11	13	14	19	23	29	37	46
Mass (kg/m)	0.222	0.395	0.616	0.888	1.579	2.466	3.854	6.313	9.864

**Table 19: Sectional areas per metre width for various bar sizes and spacing (mm<sup>2</sup>/m)**

Bar size	Spacing of bars (mm)								
	75	100	125	150	175	200	250	300	350
H6	377	283	226	188	162	141	113	94	81
H8	670	503	402	335	287	251	201	168	144
H10	1047	785	628	524	449	393	314	262	224
H12	1508	1131	905	754	646	565	452	377	323
H16	2681	2011	1608	1340	1149	1005	804	670	574
H20	4189	3142	2513	2094	1795	1571	1257	1047	898
H25	6545	4909	3927	3272	2805	2454	1963	1636	1402
H32	10723	8042	6434	5362	4596	4021	3217	2681	2298
H40	16755	12566	10053	8378	7181	6283	5027	4189	3590



# REINFORCED CONCRETE & BASIC RESTRAINED STEEL BEAM DESIGN

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‘Course B’ Module 2

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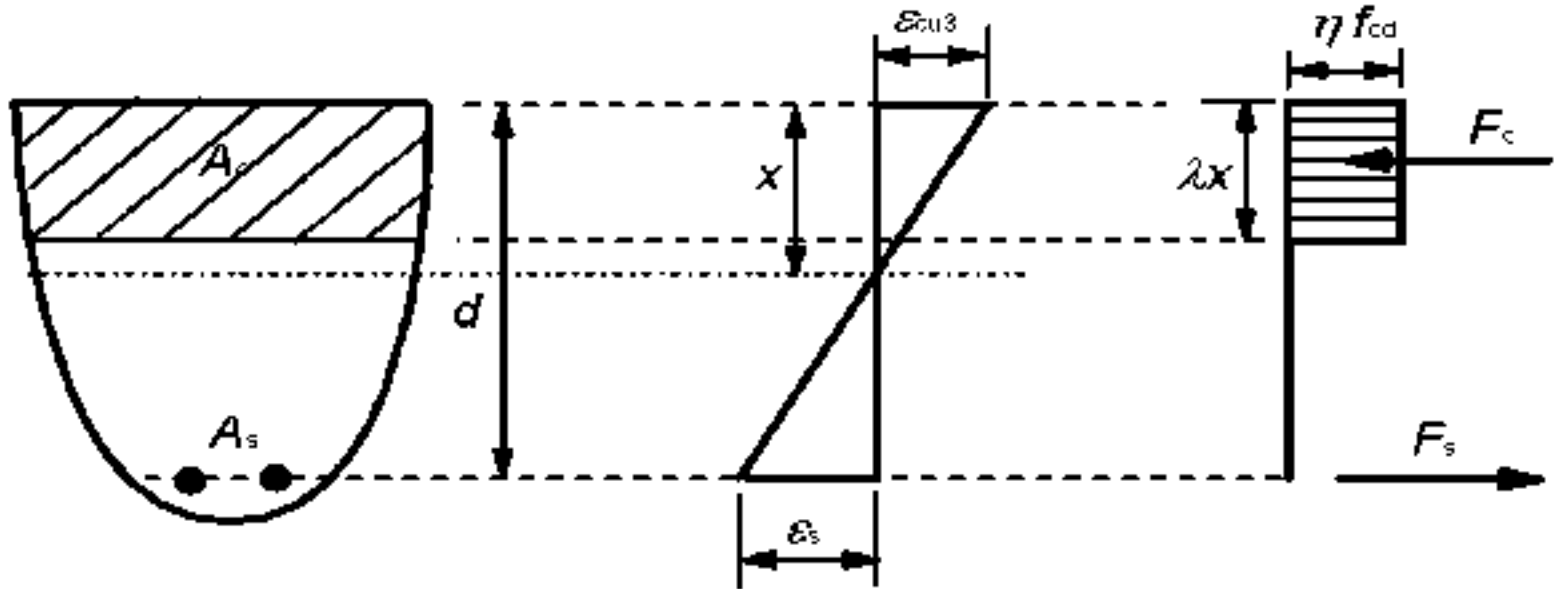
## DESIGN OF CONCRETE ELEMENTS

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2015

# Figure 1 - FLEXUAL MEMBERS – IN BENDING – A

## Simplified concrete design stress block



$$\lambda = 0,8 \quad \text{for } f_{ck} \leq 50 \text{ MPa}$$

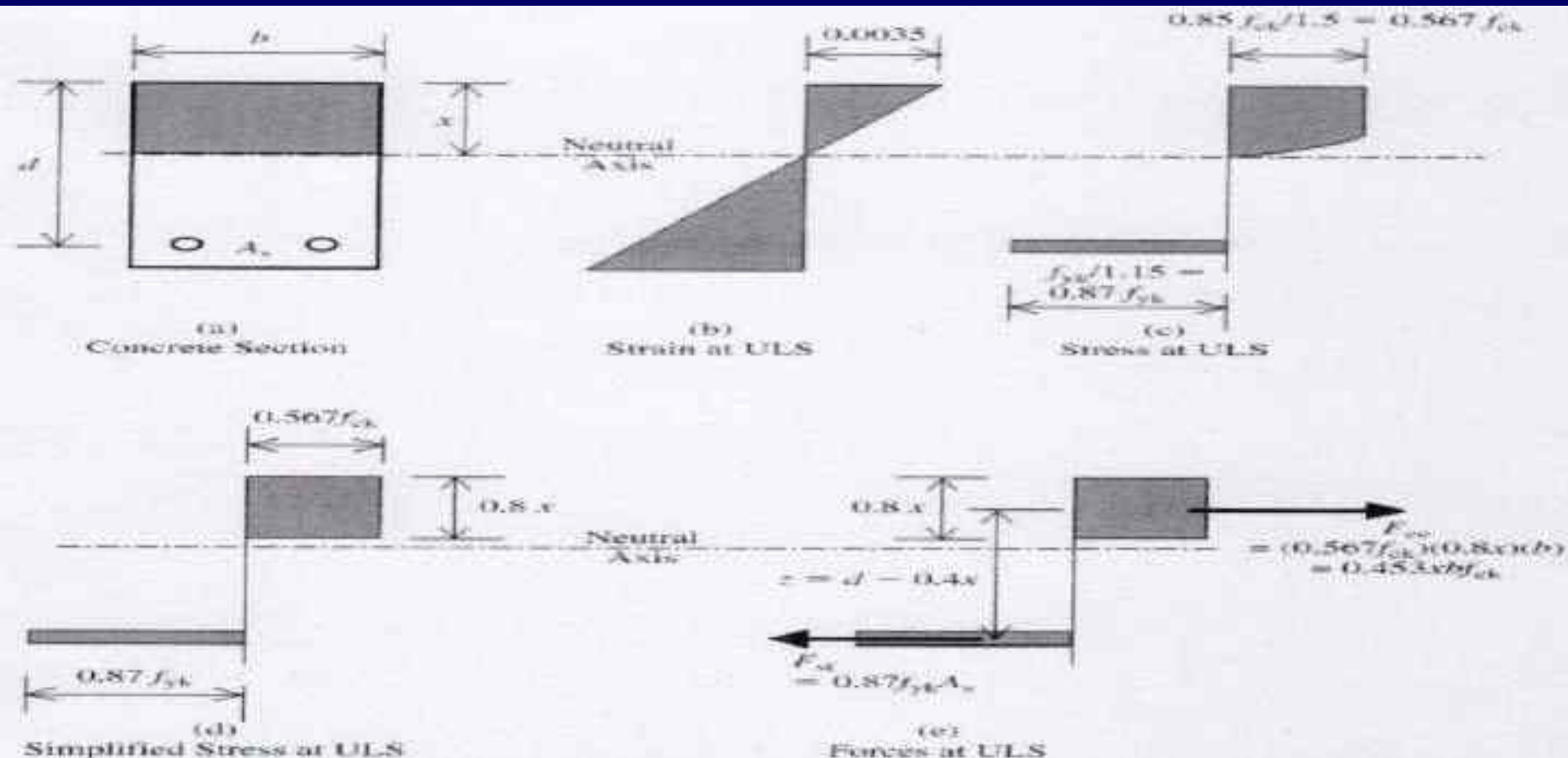
$$= 0,8 - \frac{(f_{ck} - 50)}{400} \quad \text{for } 50 < f_{ck} \leq 90 \text{ MPa}$$

$$\eta = 1,0 \quad \text{for } f_{ck} \leq 50 \text{ MPa}$$

$$= 1,0 - (f_{ck} - 50)/200 \quad \text{for } 50 < f_{ck} \leq 90 \text{ MPa}$$

# Figure 2 - FLEXURAL MEMBERS – IN BENDING – B

## Stresses and forces in a rectangular concrete beam at ULS





# Figure 3 - BENDING DESIGN

Factors for NA depth ( $n$ ) and lever arm ( $=z$ ) for concrete grade  $\leq 50$  MPa

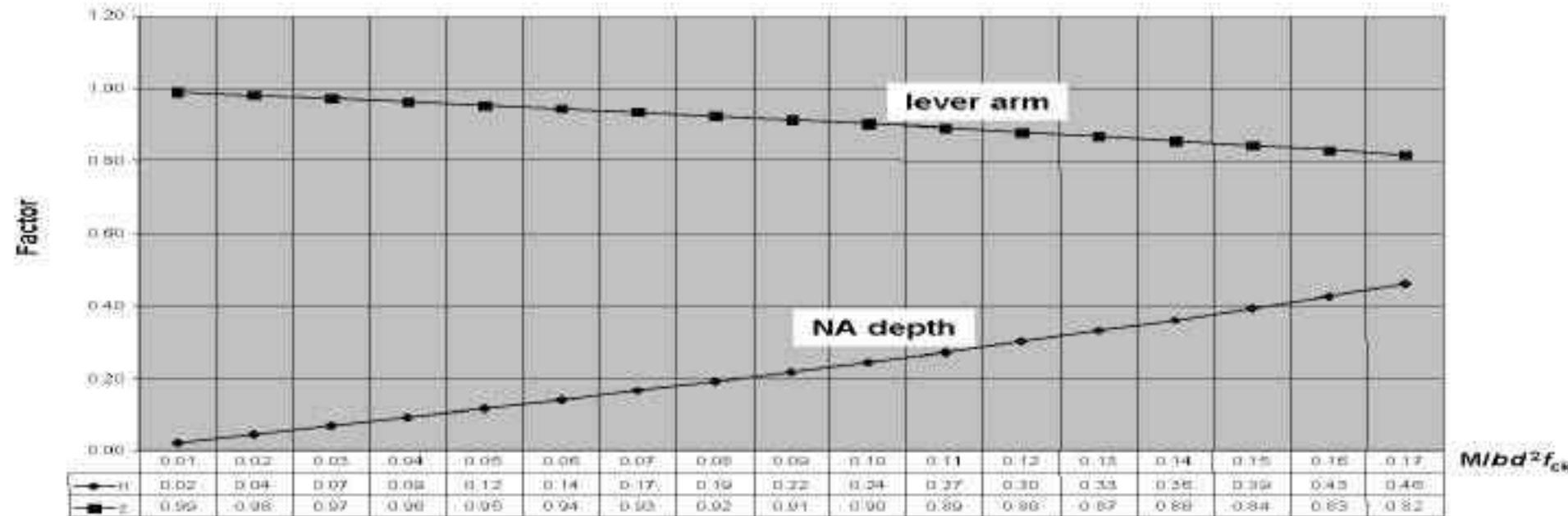


Table 1: Values of  $z/d$  for design of bending reinforcement in beams and slabs

K	up to 0.05	0.06	0.07	0.08	0.09	0.10	0.11	0.12	0.13	0.14	0.15	0.16	0.167
$z/d$	0.950	0.944	0.934	0.924	0.913	0.902	0.891	0.880	0.868	0.856	0.843	0.830	0.820
$x/d$	0.125	0.140	0.165	0.191	0.217	0.245	0.272	0.301	0.331	0.361	0.393	0.425	0.449

Formulas:  $z/d = 0.5 (1 + \sqrt{1 - 3.53K})$  but not more than 0.95

$x/d = 2.5 (1 - z/d)$

- Find  $z = d(z/d)$ .
- Find  $A_s = M/(0.87z f_{yk})$ .
- Check that  $A_s$  is not less than the minimum percentage in Module 1-B.
- Choose some reinforcing bars with an area of at least  $A_s$ .
- Check that the area of bars provided does not exceed the maximum percentage in Module 1-B. If it does then a larger beam may be needed.



# DESIGN OF BEAMS/SLABS FOR DEFLECTION SLS

Deflections of such a magnitude that members appear visibly to sag will upset the owners or occupiers of structures. It is generally accepted that a deflection larger than  $\text{span}/250$  should be avoided from the appearance point of view. A survey of structures in Germany that had given rise to complaints produced 50 examples. The measured sag was less than  $\text{span}/250$  in only two of these.

Done in 2 ways – Calculation or Tabulated Values

# SLS – CONTROL OF DEFLECTIONS

## Tabulated Values:-

**Table 2: Basic ratios of span/effective depth for simply supported beams and slabs**

Percentage of main reinforcement $\rho_t$	up to 0.35%	0.4%	0.5%	0.6%	0.7%	0.8%	0.9%	1.0%	1.1%	1.2%	1.3%	1.4%	1.5% or more
Basic span/effective depth ratio	30.0	26.6	20	19.4	18.8	18.2	17.6	17.0	16.4	15.8	15.2	14.6	14

Note that the factors vary according to the percentage of reinforcement in the beam. Generally a beam with a higher percentage of reinforcement  $\rho$  will have a deeper stress block of concrete in compression which will cause more curvature of the beam and more deflection so the limiting span to effective depth ratio limits are lower for such a beam.

For other than simple support add a K factor of 1.5 for interior span condition, 1.3 for end span condition & 0.4 for cantilevers.

When more reinforcement is provided than is required by the ULS, multiply factor by  $A_{s,prov}/A_{z,req}$

# PRINCIPLES OF SHEAR CONTROL IN EC2

Until a certain shear force  $V_{Rd,c}$  no calculated shear reinforcement is necessary (only in beams minimum shear reinforcement is prescribed)

If the design shear force is larger than this value  $V_{Rd,c}$  shear reinforcement is necessary for the full design shear force. This shear reinforcement is calculated with the variable inclination truss analogy. To this aim the strut inclination may be chosen between two values (recommended range  $1 \leq \cot \theta \leq 2,5$ )

The shear reinforcement may not exceed a defined maximum value to ensure yielding of the shear reinforcement

# CONCRETE BEAMS REINFORCED IN SHEAR (strut & tie modelling)

In a reinforced concrete beam with vertical links, shear forces are considered to be carried by the links in tension acting with diagonal concrete struts in compression, as shown in Figure 4 below:-

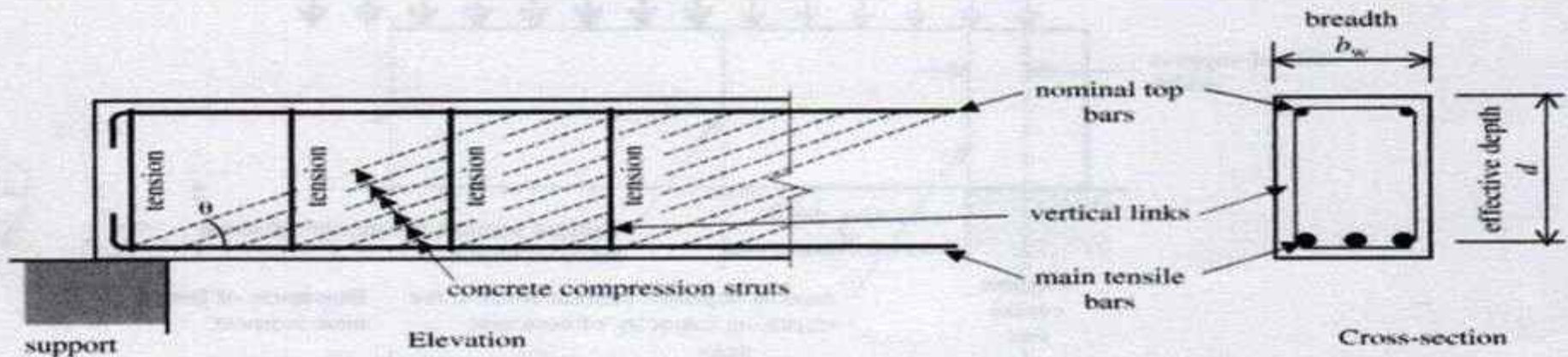
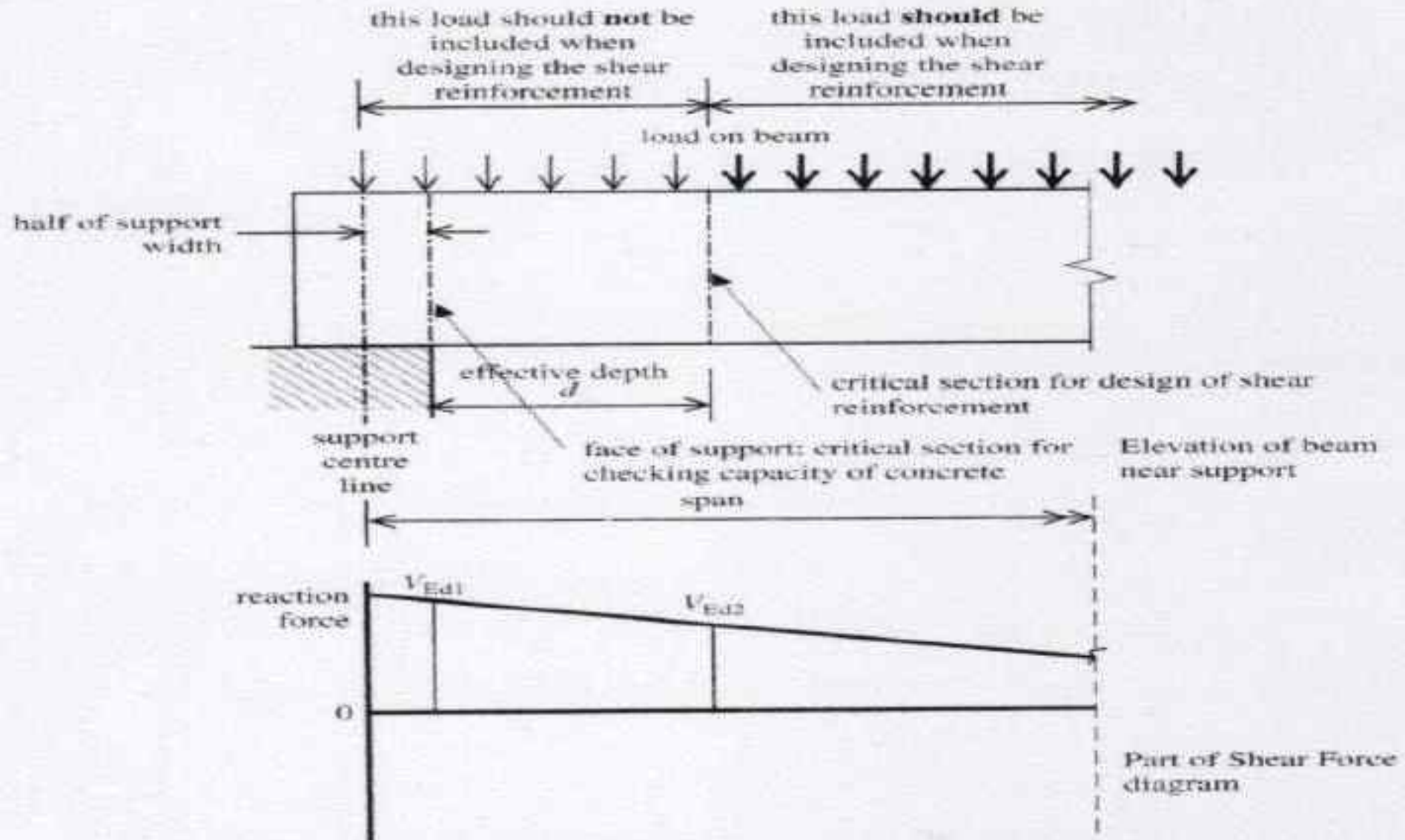


Figure 4: Beam carrying shear: links in tension and concrete in compression

EC2 allows the designer to vary the angle  $\theta$  of the strut to obtain the most economical solution. However an angle  $\theta$  of  $22^\circ$  leads to low shear reinforcement (the minimum allowed in EC2) will give practical designs in most cases. The highest angle of  $45^\circ$  leads to thin webs.

# Figure 5 – Critical sections for a beam carrying shear





# PROCEDURE FOR CALCULATING SHEAR LINKS - 1

1. Find  $V_{Ed1}$  = shear force at the face of the support.
2. Find  $v_{Ed1}$  = shear stress at the face of the support =  $V_{Ed1}/(0.9b_wd)$ .
3. Find the concrete strut capacity  $v_{Rd}$  from Table 3.
4. If  $v_{Ed1}$  is greater than  $v_{Rd}$  then see EC2 for use of other values of  $\theta$  up to  $45^\circ$ . In some cases it may be necessary to use a larger beam or a higher class of concrete.
5. If  $v_{Ed1}$  is not greater than  $v_{Rd}$  then use  $\theta = 22^\circ$  and follow steps 6–11.
6. Find  $V_{Ed2}$  = shear force at a distance  $d$  from the face of the support.
7. Find  $v_{Ed2}$  = shear stress at a distance  $d$  from the face of the support =  $V_{Ed2}/(0.9b_wd)$ .
8. Calculate the area of shear reinforcement required:  $A_{sw}/s = 0.4v_{Ed2}b_w/0.87f_{yk}$ .  
Since  $f_{yk}$  is always  $500 \text{ N/mm}^2$ , this gives  $A_{sw}/s = 0.00092v_{Ed2}b_w$ .
9. Find the minimum  $A_{sw}/s$  from Table 4
10. Consider the following limits to the spacing of the links along the beam:  
Minimum spacing                      75 mm  
Maximum spacing                       $0.75d$  but not more than 600 mm.  
Economy will be achieved by having as few links as possible, so in step 11 it is best to choose a spacing  $s$  close to the maximum permitted value.
11. Choose a link size  $A_{sw}$  and link spacing  $s$  so that  $A_{sw}/s$  is not less than the values from steps 8 and 9. Table 5.

# PROCEDURE FOR CALCULATING SHEAR LINKS - 2

**Table 3:**  $\nu_{Rd}$  concrete strut capacities for calculations of shear in beams

$f_{ck}$ (N/mm <sup>2</sup> )	25	28	30	32	35	40	45	50
$\nu_{Rd}$ (N/mm <sup>2</sup> )	3.10	3.43	3.64	3.84	4.15	4.63	5.08	5.51

Formula:  $\nu_{Rd} = 0.36(1 - f_{ck}/250)f_{ck}/(\cot\theta + \tan\theta)$  with  $\theta = 22^\circ$ .

**Table 4 :** Minimum shear reinforcement in beams

$f_{ck}$ (N/mm <sup>2</sup> )	25	28	30	32	35	40	45	50
Minimum $A_{sw}/s$	$0.0008b_w$	$0.00085b_w$	$0.00088b_w$	$0.00091b_w$	$0.00095b_w$	$0.00101b_w$	$0.00107b_w$	$0.00113b_w$

Formula: minimum ratio =  $0.08 \sqrt{f_{ck}/f_{yk}}$

**Table 5:** Area of shear links  $A_{sw}/s$  (mm<sup>2</sup>/mm) for various link sizes and spacings (based on two legs per link)

Bar size	Spacing of links (mm)								
	75	100	125	150	175	200	250	300	350
H6	0.754	0.565	0.452	0.377	0.323	0.283	0.226	0.188	0.162
H8	1.340	1.005	0.804	0.670	0.574	0.503	0.402	0.335	0.287
H10	2.094	1.571	1.257	1.047	0.898	0.785	0.628	0.524	0.449
H12	3.016	2.262	1.81	1.508	1.293	1.131	0.905	0.754	0.646
H16	5.362	4.021	3.217	2.681	2.298	2.011	1.608	1.340	1.149

# DESIGN OF SLABS FOR SHEAR – 1 ULS

1. Find  $V_{Ed}$  = shear force at the face of the supporting beam or wall.
2. Find  $v_{Ed}$  = shear stress =  $V_{Ed}/(0.9bd)$ . Normally  $b = 1000$  mm.
3. Find  $v_{Rd,c}$  from Table **6**.
4. Multiply  $v_{Rd,c}$  by the modification factor from Table **7**.
5. If  $v_{Ed}$  is not greater than the modified  $v_{Rd,c}$  then the slab is safe without shear reinforcement.



# DESIGN OF SLABS FOR SHEAR – 2 ULS

**Table 6:**  $v_{Rd,c}$  shear resistance of solid slabs in class C25/30 concrete without shear reinforcement ( $N/mm^2$ )

Reinforcement ratio $\rho_1 = A_f/bd$	Effective depth $d$ (mm)								
	200 mm or less	225	250	275	300	400	500	600	750
0.25%	0.49	0.47	0.46	0.44	0.43	0.39	0.37	0.35	0.34
0.50%	0.56	0.54	0.53	0.52	0.51	0.48	0.45	0.44	0.42
0.75%	0.64	0.62	0.60	0.59	0.58	0.54	0.52	0.50	0.48
1.00%	0.70	0.68	0.66	0.65	0.64	0.60	0.57	0.55	0.53
1.25%	0.76	0.73	0.72	0.70	0.69	0.65	0.62	0.60	0.57
1.50%	0.80	0.78	0.76	0.74	0.73	0.69	0.66	0.63	0.61
1.75%	0.85	0.82	0.80	0.78	0.77	0.72	0.69	0.67	0.64
2.00%	0.88	0.86	0.84	0.82	0.80	0.75	0.72	0.70	0.67

Formula:  $v_{Rd,c} = 0.12k(100\rho_1f_{ck})^{1/3}$  where  $k = 1 + \sqrt{(200/d)}$  but not more than 2.0.

Source: Equation 6.2a of EC2 Part 1-1.

**Table 7:** Concrete strength modification factors for use with factors from Table 6

$f_{ck}$ ( $N/mm^2$ )	25	28	30	32	35	40	45	50
Mod. factor	1.00	1.04	1.06	1.09	1.12	1.17	1.22	1.26

Formula: Modification factor =  $(f_{ck}/25)^{1/3}$ .

# Sizes and reinforcement of columns - 1

Where possible it will generally be best to use 'stocky columns' (i.e. generally for typical columns for which the ratio of the effective height to the least lateral dimension does not exceed 15) as this will avoid the necessity of designing for the effects of slenderness. Slenderness effects can normally be neglected in non-sway structures where the ratio of the effective height to the least lateral dimension of the column is less than 15. For the purpose of initial design, the effective height of a braced column may be taken as 0.85 times the storey height.

The columns should be designed as axially loaded, but to compensate for the effect of eccentricities, the ultimate load from the floor immediately above the column being considered should be multiplied by the factors listed below:

- For columns loaded by beams and/or slabs of similar stiffness on both sides of the column in two directions at right-angles to each other, e.g. Some internal columns. 1.25
- For columns loaded in two directions at right-angles to each other by unbalanced beams and/or slabs, e.g. corner columns. 2.00
- In all other cases, e.g. façade columns. 1.50

# Sizes and reinforcement of columns - 2

It is recommended that the columns are made the same size through at least the two topmost storeys, as the above factors may lead to inadequate sizes if applied to top storey columns for which the moments tend to be large in relation to the axial loads.

For the initial design of columns, the required cross-sectional area may be calculated by dividing the ultimate load by the selected equivalent 'stress' given in Table 8. Alternatively, for a known column size the ultimate load capacity may be found by using the selected equivalent 'stress'.

When choosing the column dimensions, care should be taken to see that the column remains stocky, as defined above.

# Sizes and reinforcement of columns - 3

Table 8 Equivalent stress values			
Reinforcement (500MPa) percentage $\rho$	Equivalent stresses (MPa) for concrete strength classes		
	C25/30	C30/37	C35/45
$\rho = 1\%$	14	17	19
$\rho = 2\%$	18	20	22
$\rho = 3\%$	21	23	25
$\rho = 4\%$	24	27	29

The equivalent 'stresses' given in Table 8 are derived from the expression:

$$\text{stress} = 0.44 f_{ck} + \frac{\rho}{100} (0.67 f_{yk} - 0.44 f_{ck})$$

Where:  $f_{ck}$  is the characteristic concrete strength in MPa  
 $f_{yk}$  the characteristic strength of reinforcement in MPa  
 $\rho$  the percentage of reinforcement.

Where slender columns (i.e. the ratio of the effective height  $l_0$  to the least lateral dimension,  $b$ , exceeds 15) are used, the ultimate load capacity of the column or equivalent 'stress' should be reduced by the appropriate factor from Figure 4.1. In braced frames  $l_0$  may be taken as the clear floor to soffit height.

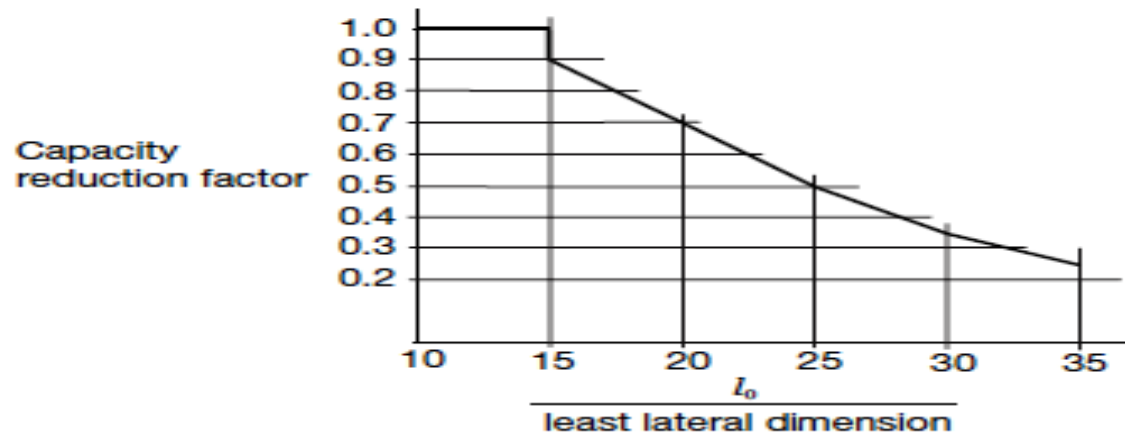
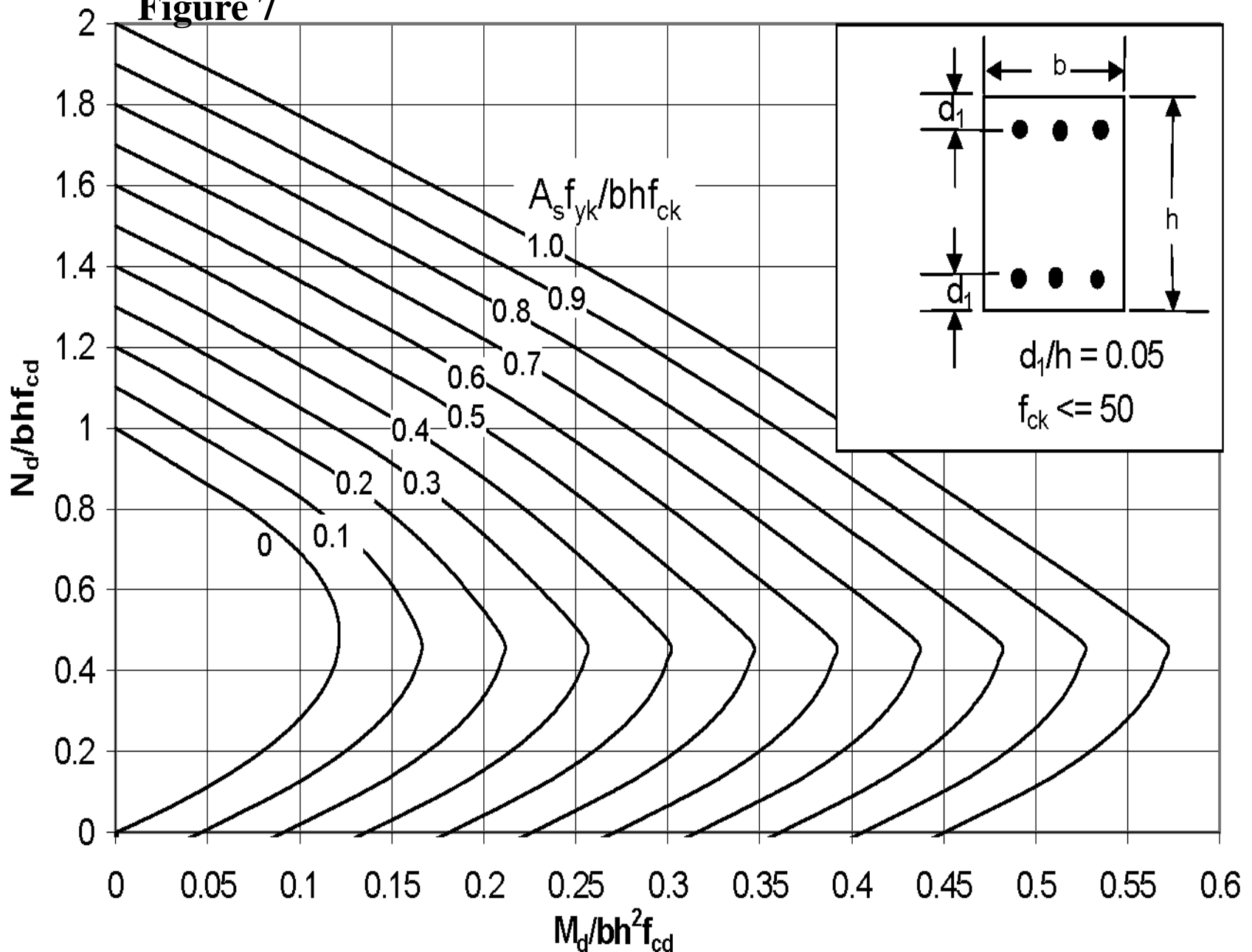


Figure 6 - Reduction factors for slender columns

# Column design chart for $f_{ck} \leq 50$ MPa

Figure 7



# DEMYSTIFYING THE EUROCODES

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## ‘Course B’ Module 3

### - preliminary sizing to reinforced concrete & steel members

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12<sup>th</sup> /14<sup>th</sup>  
May  
2015

# DEMYSTIFYING THE EUROCODES

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## ‘Course B’ Module 4

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## DESIGN EXAMPLES IN REINFORCED CONCRETE

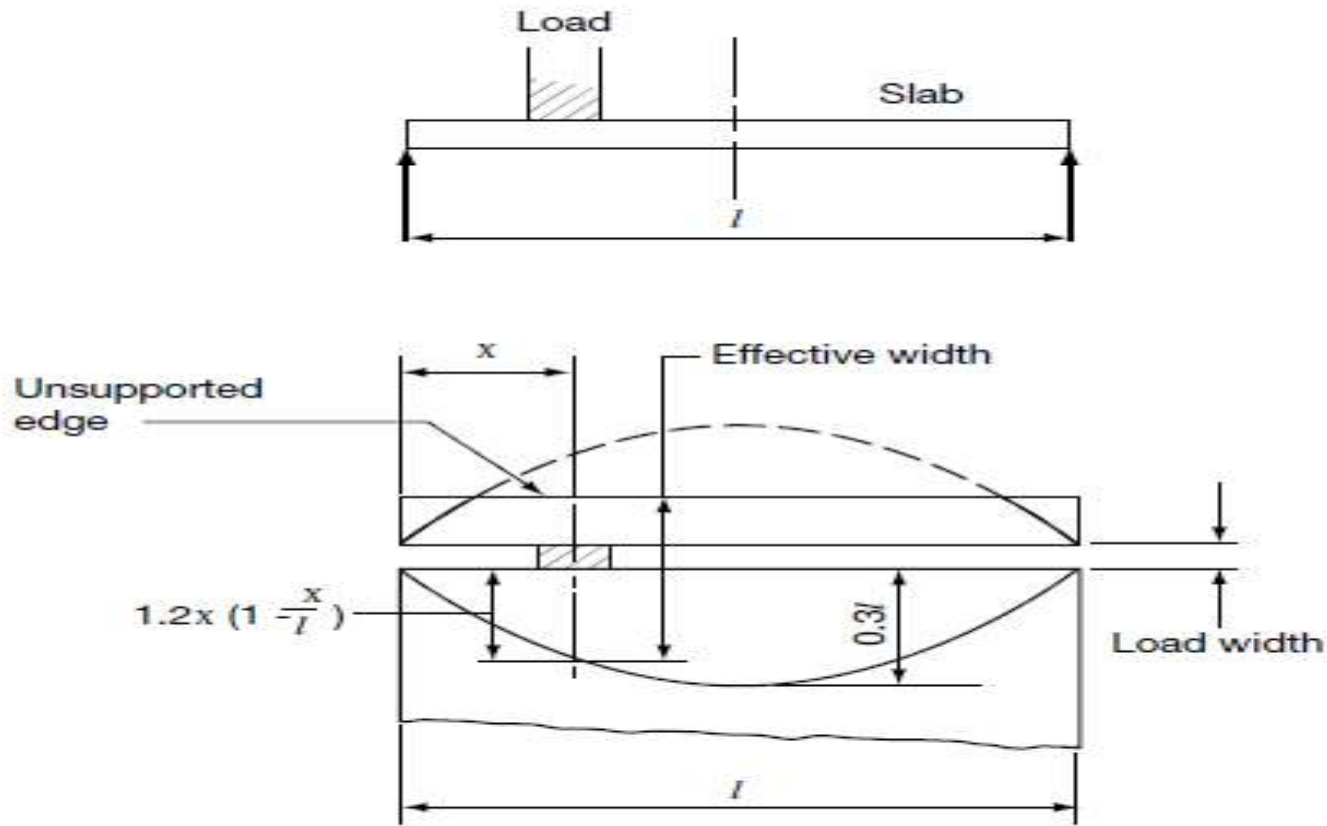
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Ref.	Calculations	Output																
	<p>A simply supported RC slab has an effective span of 5m (preliminary stirrups 175mm)</p> <p>Exposure XC1 - interior of bldgs</p> <p>For XC1 C20/25 0.7 <math>240 \text{ kg/m}^3</math> - 25mm cover</p> <p>Fire resistance 1h min thickness 80mm min ans 20mm</p> <p><b>SLAB Loading - STRUCT</b></p> <table> <tr> <td>self-wt</td> <td>0.175 x 25</td> <td><math>\Rightarrow</math></td> <td>5.91</td> </tr> <tr> <td>Finish</td> <td>0.1 x 18</td> <td><math>\Rightarrow</math></td> <td>2.43</td> </tr> <tr> <td>L.L</td> <td>1.5</td> <td><math>\Rightarrow</math></td> <td>2.25</td> </tr> <tr> <td></td> <td></td> <td></td> <td><u>10.59 kN/m<sup>2</sup></u></td> </tr> </table> <p>BM <math>\rightarrow 10.59 \cdot 5^2 / 8 \Rightarrow 33.09 \text{ kN-m}</math></p> <p>The Concrete Centre spreadsheet yields</p> <p><math>A_s(\text{req}) \rightarrow 617 \text{ mm}^2/\text{m}</math></p> <p><math>A_s(\text{min}) \rightarrow 423 \text{ mm}^2/\text{m}</math></p> <p><math>A_s(\text{prov}) \rightarrow 785 \text{ mm}^2/\text{m}</math></p> <p>Defl<sup>n</sup> check (span/250 condition)</p> <p>Permissible <math>L/d \Rightarrow 36.43</math></p> <p>Actual <math>L/d \Rightarrow 36.48 \checkmark</math></p>	self-wt	0.175 x 25	$\Rightarrow$	5.91	Finish	0.1 x 18	$\Rightarrow$	2.43	L.L	1.5	$\Rightarrow$	2.25				<u>10.59 kN/m<sup>2</sup></u>	<p>(table 6)</p> <p>(table 7)</p> <p>(table 8)</p> <p>ECD equibid</p>
self-wt	0.175 x 25	$\Rightarrow$	5.91															
Finish	0.1 x 18	$\Rightarrow$	2.43															
L.L	1.5	$\Rightarrow$	2.25															
			<u>10.59 kN/m<sup>2</sup></u>															





**Fig 1:** Effective width of solid slab carrying a concentrated load near an unsupported edge

For precast concrete construction the width of slab should not exceed the width of the loaded area +the width of 3 precast units, when there is no topping or the width of 4 units where topping is at least 30mm thick. In no case should the width be taken as extending more than  $0.25L$  on either side of the loaded area.

# SPAN: DEPTH RATIOS FOR CONCRETE STRUCTURES

In the case of reinforced concrete the span:deflection ratio is taken over by span to depth ratios. The span-to-depth ratios of 20 for simply supported spans is based on a span-to-deflection ratio of 1:250.

From bending theory:

$$\text{Span/depth: } L/d = 4.8 * E / ( f_{ck} * q )$$

Where  $q$  is the allowable span to deflection factor, which for 1/500 works out at:


$$\text{Span/depth} = 4.8 * 28 \text{ kN/mm}^2 / (25\text{N/mm}^2 * 500) = 10.75$$

This as compared to the conventional 20 specified in EC2.

Ref.	Calculations	Output									
	<p>Go to LOAD ANALYSIS WORKSHEET CoomA/Module 4. TRIANGULATED loading from overlying 4 flrs. <math>(21 + 36.75 + 63) + 37.8 \cdot 1.35 \Rightarrow 171.78 \text{ kN/m}</math> this line load is spread over <math>2.0 \cdot 3.6 \text{ m} + 0.225 \text{ m} \Rightarrow 3.825 \text{ m}</math> u.d.l.: <math>171.78 \text{ kN/m} / 3.825 \text{ m} \Rightarrow 45 \text{ kN/m}^2</math> BM <math>\Rightarrow 45 \cdot 6^2 / 8 \Rightarrow 202.5 \text{ kN-m/m}</math></p> <p>The Concrete Control spreadsheet yields <math>A_c(\text{req}) \Rightarrow 1,712 \text{ mm}^2/\text{m}</math> <math>A_c(\text{act}) \Rightarrow 1,712 \text{ mm}^2/\text{m}</math> <math>A_c(\text{prov}) \Rightarrow 2,011 \text{ mm}^2/\text{m}</math></p> <p>Defl check (brittle finishes/partitions - span/500) Permissible <math>L/d \Rightarrow 28.15</math> Actual <math>L/d \Rightarrow 14.22 &gt; 2x \checkmark</math></p> <p>Shear (VLS) - check <math>V_{Ed} = (45 + 0.46 \cdot 25 \text{ kN/m}^2 \cdot 1.35) \cdot 3 \text{ m} \Rightarrow 180 \text{ kN/m}</math> <math>V_{red} = V_{Ed} / 0.9bd \Rightarrow 180 / 0.9 \cdot 1.422 \Rightarrow 0.474 \text{ N/mm}^2</math> <math>c = A_s / bd \Rightarrow 2011 / 1.422 \Rightarrow 0.476\%</math></p> <table border="0"> <tr> <td>TABLES</td> <td><math>d = 400</math></td> <td><math>d = 500</math></td> </tr> <tr> <td><math>f_1 0.25\%</math></td> <td>0.37</td> <td>0.37</td> </tr> <tr> <td><math>f_1 0.30\%</math></td> <td>0.48</td> <td>0.45</td> </tr> </table> <p>by interpolation with <math>d = 422</math> <math>c = 0.476\%</math> <math>V_{red} \Rightarrow 0.46 \text{ N/mm}^2</math></p>	TABLES	$d = 400$	$d = 500$	$f_1 0.25\%$	0.37	0.37	$f_1 0.30\%$	0.48	0.45	
TABLES	$d = 400$	$d = 500$									
$f_1 0.25\%$	0.37	0.37									
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job title: REINFORCED CONCRETE DESIGN

Ref.	Calculations	Output																														
<p>ECO equ 6.10</p>	<p>Design of an internal column on a structural grid of 8m x 6m with column face limited to 0.25m. Office loading to be taken on a flat slab construction. From module 3 slab thickness on 8m span to be 250mm thick.</p> <p><u>Loadings</u></p> <table border="0"> <tr> <td>self wt</td> <td>0.25</td> <td>25</td> <td><math>\frac{135}{1000}</math></td> <td><math>\rightarrow</math></td> <td>8.44 kN/m<sup>2</sup></td> </tr> <tr> <td>finish</td> <td>0.1</td> <td>18</td> <td><math>\frac{135}{1000}</math></td> <td><math>\rightarrow</math></td> <td>2.43 kN/m<sup>2</sup></td> </tr> <tr> <td>D.L</td> <td>2.5</td> <td></td> <td><math>\frac{15}{1000}</math></td> <td><math>\rightarrow</math></td> <td>3.75 kN/m<sup>2</sup></td> </tr> <tr> <td>movable partitions</td> <td>1.0</td> <td></td> <td><math>\frac{15}{1000}</math></td> <td><math>\rightarrow</math></td> <td>1.50 kN/m<sup>2</sup></td> </tr> <tr> <td></td> <td></td> <td></td> <td></td> <td></td> <td><u>16.12 kN/m<sup>2</sup></u></td> </tr> </table>	self wt	0.25	25	$\frac{135}{1000}$	$\rightarrow$	8.44 kN/m <sup>2</sup>	finish	0.1	18	$\frac{135}{1000}$	$\rightarrow$	2.43 kN/m <sup>2</sup>	D.L	2.5		$\frac{15}{1000}$	$\rightarrow$	3.75 kN/m <sup>2</sup>	movable partitions	1.0		$\frac{15}{1000}$	$\rightarrow$	1.50 kN/m <sup>2</sup>						<u>16.12 kN/m<sup>2</sup></u>	
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<p>equ 6.10 a equ 6.10 b</p>	<p><math>(8.44 + 2.43) + 0.7(3.75 + 1.5) \rightarrow 14.55 \text{ kN/m}^2</math>  <math>0.85(8.44 + 2.43) + (3.75 + 1.5) \rightarrow 14.49 \text{ kN/m}^2</math></p>																															
	<p><u>Design for Col at section A-A 0.25m x ?</u></p> <p>as this is a braced building  <math>l_c/h \Rightarrow 15</math>  <math>h \Rightarrow (0.85 \times 3.5) / 15 \Rightarrow 0.198\text{m} &lt; 0.25\text{m} \checkmark</math></p> <p>For concrete grade C25/30 with 2% rebar  <math>f_{eqv} \Rightarrow 18 \text{ N/mm}^2</math></p> <p><math>N \Rightarrow (14.55 \text{ kN/m}^2 \times 6\text{m} \times 8\text{m}) \times 1.25 \times 4 \text{ flo} \Rightarrow 3480 \text{ kN}</math>  <math>N \Rightarrow f_{eqv} b h</math>  <math>b \Rightarrow 3,480 \text{ kN} / 18,000 \text{ kN/m}^2 / 0.25 \Rightarrow 0.773\text{m}</math></p>	<p>Col size 0.25 x 0.775m</p>																														

Ref.	Calculations	Outputs
	<p><u>Design for Col. at Section B-B</u></p> <p><math>l_e/h \Rightarrow 15</math>  <math>h \Rightarrow (0.85 \times 6) / 0.25 \Rightarrow 20.4 &gt; 15</math> (slender)                      capacity reduction factor  <math>\alpha \Rightarrow 0.7</math></p> <p>For concrete grade C25/30 with 4% airt.  <math>f_{eqv} \Rightarrow 24 \text{ N/mm}^2</math>  <math>N \Rightarrow (14.5 \text{ kN/m}^2 \times 6 \text{ m} \times 6 \text{ m}) \times 1.25 \times SF1 \Rightarrow 4,350 \text{ kN}</math>  <math>N \Rightarrow \alpha f_{eqv} b h</math>  <math>b \Rightarrow 4,350 \text{ kN} / (24,000 \text{ kN/m}^2 / 0.25) \Rightarrow 1,041 \text{ mm}</math></p> <p>check from col chart <math>F_{ck} \leq 50 \text{ MPa}</math>  <math>N / b h f_{cd} \Rightarrow 3,480 / (0.25 \times 77.5 \times 25 / 1.5) \Rightarrow 1.078</math>  <math>M_{non} \Rightarrow 0.05 h_{min} \cdot N</math>  <math>\Rightarrow 0.05 \cdot 0.25 \cdot 3,480 \Rightarrow 43.5 \text{ kN-m}</math>  <math>M / b h^2 \cdot f_{cd} \Rightarrow 43.5 / (0.25 \cdot 77.5 \cdot 77.5 \cdot 25 / 1.5) \Rightarrow 0.0173</math>  <math>A_s f_{yk} / b h F_{ck} \Rightarrow 0.15</math>  <math>A_s / b h \Rightarrow 0.15 \cdot 25 / 500 \Rightarrow 0.75\%??</math></p>	<p>Col 5120  <math>1.0 \text{ m} \times 0.25 \text{ m}</math></p>



# Example:- Beam design for bending ULS, deflection SLS and shear ULS

A simply supported reinforced concrete beam 650 mm deep and 300 mm wide has an effective span of 8.50 m onto supports which are 300 mm wide. In addition to its own self-weight, the beam carries the following loads:

Dead load	22 kN/m
Imposed load	17 kN/m.

The beam is in grade C35/45 concrete and will be inside a building where a fire resistance of 1 hour is required. The main reinforcing bars are size H32 and the links are size H12.

Determine the reinforcement required in the beam and check whether the deflection of the beam will be acceptable.

# Calculations for The Example - 1

<b>Data given</b>	
Beam width $b$	$b = 300 \text{ mm}$
Beam overall height $h$	$h = 650 \text{ mm}$
Effective span $L$	$L = 8.5 \text{ m}$
Dead load excluding beam SW	$g_k = 22.0 \text{ kN/m}$
Imposed load	$q_k = 17.0 \text{ kN/m}$
Concrete grade	C35/45
<b>Cover to bars</b>	
Fire resistance: From Table 9/1, with a beam width of 300 mm	Min. axis distance = 25 mm
Durability: From Table 6/1, exposure class is XC1. From Table 7/1,	Min. cover to all bars = 25 mm
Placing of concrete: Min. cover to H32 bars = 32 + 10	Min. cover to main bars = 42 mm
Min cover to H12 links = 12 + 10	Min. cover to links = 22 mm
These three can be achieved by specifying a cover of 30 mm to the links, which will give 30 + 12 = 42 mm cover to the main bars and 30 + 12 + 32/2 = 58 mm axis distance to the main bars	Provide 30 mm cover to all bars
<b>Material strengths</b>	
$f_{ck}$ (characteristic cylinder strength of concrete)	$f_{ck} = 35 \text{ N/mm}^2$
$f_{yk}$ (characteristic tensile strength of reinforcement, class H)	$f_{yk} = 500 \text{ N/mm}^2$
<b>Loading</b>	
Unit weight of concrete = 25 kN/m <sup>3</sup> , so beam self-weight = 0.30 × 0.65 × 25 × 8.5	Beam self-weight = 41.4 kN
Total permanent load = 41.4 + 22.0 × 8.5	$G_k = 228.4 \text{ kN}$
Total imposed load = 17.0 × 8.5	$Q_k = 144.5 \text{ kN}$
Using $\gamma_f = 1.35$ for permanent loads and $\gamma_q = 1.50$ for variable loads, ultimate load	Ultimate load
$F = 1.35G_k + 1.50Q_k = 1.35 \times 228.4 + 1.50 \times 144.5$	$F = 525.1 \text{ kN}$
<b>Bending ULS</b>	
$M_u = FL/8 = 525.1 \times 8.5/8$	$M_u = 557.9 \text{ kNm} = 557.9 \times 10^6 \text{ Nmm}$
With H12 links, H32 main bars and cover to all bars of 30 mm	
Effective depth $d = 650 - 30 - 12 - 32/2$	$d = 592 \text{ mm}$
$K = M_u/bd^2f_{ck} = 557.9 \times 10^6 / (300 \times 592^2 \times 35)$	$K = 0.152$
$K$ should not be more than 0.167	Accept
$z/d = 0.5(1 + \sqrt{1 - 3.53K}) = 0.5(1 + \sqrt{1 - 3.53 \times 0.152})$	$z/d = 0.840$
$z/d$ not more than 0.95	Accept
$z = d(z/d) = 592 \times 0.840$	$z = 497.3 \text{ mm}$



# Calculations for The Example - 2

$$A_s = M / (0.87 f_{yk}) = 557.9 \times 10^6 / (0.87 \times 497.3 \times 500)$$

From Table 11/1 min.  $A_s = 0.17\%$  of  $bh = 0.17 \times 300 \times 650 / 100$

From Table 19/1 choose bars to provide at least 2579 mm<sup>2</sup>

$$A_s = 2579 \text{ mm}^2$$

$$\text{Min. } A_s = 332 \text{ mm}^2$$

Use 2no. H32 bars plus 2no. H25 bars

$$A_{s, \text{prov}} = 2590 \text{ mm}^2$$

Percentage of reinforcement =  $A_{s, \text{prov}} \times 100 / bhd = 2590 \times 100 / (300 \times 592)$   
 = 1.46%, which is less than 4% from Table 14/1.

Accept

## Span/effective depth ratio calculations for deflection SLS

Percentage of main reinf.  $\rho_1 = 100 A_{s, \text{req}} / bhd = 100 \times 2579 / (300 \times 592)$

$$\rho_1 = 1.45\%$$

From Table 2/2 by interpolation

$$\text{modification factor} = 2590 / 2579$$

$$\text{Basic ratio} = 14.3$$

$$\text{Mod. factor} = 1.004$$

Modification factor should not be more than 1.5

Accept

Permitted ratio = basic ratio  $\times$  mod. factor =  $14.3 \times 1.004$

$$\text{Permitted ratio} = 14.4$$

Actual ratio = span/effective depth =  $8.5 / 0.592$

$$\text{Actual ratio} = 14.4$$

Actual ratio is not more than permitted ratio

Accept

Check whether two No. H32 + two No. H25 bars will fit in the width of the beam

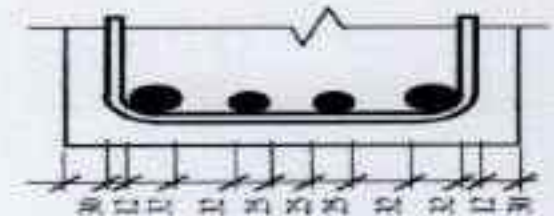
Minimum gaps between bars = bar size

The sketch opposite shows bar sizes, cover and minimum bar spacing.

The minimum beam width required

$$= 30 + 12 + 32 + 32 + 25 + 25 + 25 + 32 + 32 + 12 + 30$$

$$= 287 \text{ mm}$$



Min. beam width = 287 mm

Accept

Check maximum bar spacing

From Note 2 of Table 12/1

$$\text{steel stress} = 435(G_k + 0.8Q_k) / (1.35G_k + 1.50Q_k)$$

$$= 435(228.4 + 0.8 \times 144.5) / (1.35 \times 228.4 + 1.50 \times 144.5)$$

$$\text{Steel stress} = 285 \text{ N/mm}^2$$

From Table 12/1 since the bar size is more than 12 mm, we must meet the requirement for maximum bar spacing:

- If the steel stress was 280 N/mm<sup>2</sup> the maximum spacing would be 150 mm
- If the steel stress was 320 N/mm<sup>2</sup> the maximum spacing would be 100 mm

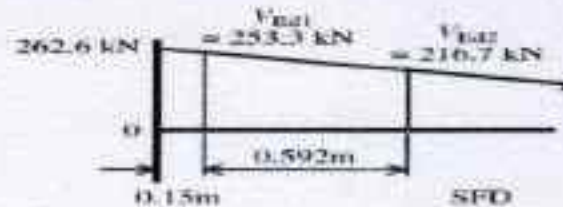
By interpolation, a steel stress of 285 N/mm<sup>2</sup> gives a maximum spacing of 144 mm

Accept

# Calculations for The Example - 3

## Shear ULS

The figure below shows part of the Shear Force diagram. The values in the diagram are calculated below.



As beam and loading are symmetric, reaction =  $F/2 = 525.1/2$

Width of support = 300 mm, so distance from centre of support to face of support =  $0.30/2$

$$V_{Ed1} = 262.6 - 525.1 \times 0.15/8.5$$

$$v_{Ed1} = V_{Ed1}/(0.9b_wd) = 253.3 \times 10^3 / (0.9 \times 300 \times 592)$$

From Table 3/2, with  $f_{ck} = 35 \text{ N/mm}^2$

Check  $v_{Ed1}$  is not more than  $v_{Rd1}$

Effective depth  $d = 0.592 \text{ m}$ , so  $V_{Ed2} = 253.3 - 525.1 \times 0.592/8.5$

$$v_{Ed2} = V_{Ed2}/(0.9b_wd) = 216.7 \times 10^3 / (0.9 \times 300 \times 592)$$

$$A_{sv}/s = 0.00092v_{Ed2}b_w = 0.00092 \times 1.36 \times 300$$

From Table 4/2, with  $f_{ck} = 35 \text{ N/mm}^2$ ,

$$\text{min. } A_{sv}/s = 0.00095b_w = 0.00095 \times 300$$

Max. link spacing =  $0.75d = 0.75 \times 592$

From Table 5/2, with  $A_{sv}/s$  not less than  $0.375 \text{ mm}^2/\text{mm}$

Reaction = 262.6 kN

Dist. = 0.15 m

$$V_{Ed1} = 253.3 \text{ kN} = 253.3 \times 10^3 \text{ N}$$

$$v_{Ed1} = 4.58 \text{ N/mm}^2$$

$$v_{Rd1} = 4.15 \text{ N/mm}^2$$

Accept

$$V_{Ed2} = 216.7 \text{ kN}$$

$$v_{Ed2} = 1.36 \text{ N/mm}^2$$

$$A_{sv}/s = 0.375 \text{ mm}^2/\text{mm}$$

Min.  $A_{sv}/s = 0.29 \text{ mm}^2/\text{mm}$

Max. link spacing = 444 mm

Use H10 links at 400 mm centres

(see Figure 3.15)

$$A_{sv}/s = 0.393 \text{ mm}^2/\text{mm}$$

5/2.

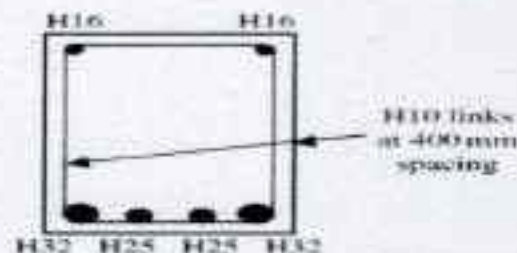


Figure 2:- Cross-section of beam in Example

# DEMYSTIFYING THE EUROCODES

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## ‘Course B’ Module 5

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## DESIGN OF A 12m RESTRAINED STEEL BEAM IN A RETAIL OUTLET

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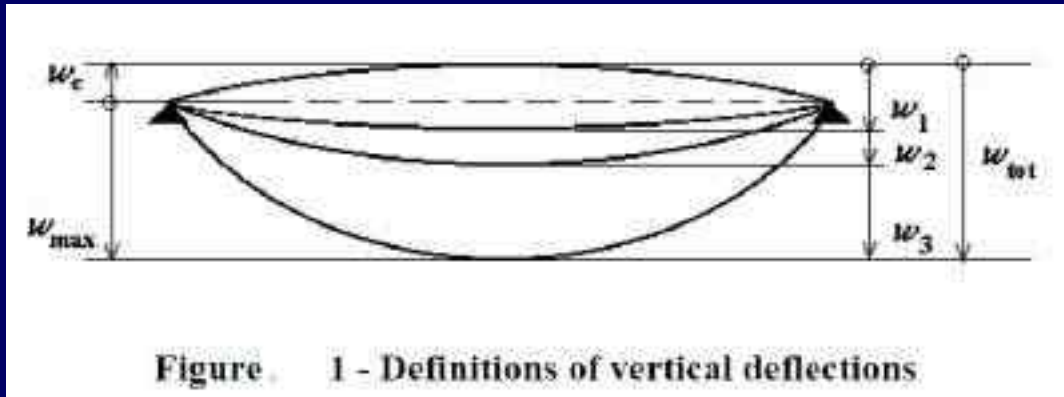
12<sup>th</sup> /14<sup>th</sup>  
May  
2015



# SERVICEABILITY LIMIT STATES FOR BUILDINGS EC3 - 1

## Vertical deflections

With reference to EN 1990 – Annex A1.4 limits for vertical deflections according to Figure 1 below:-



If the functioning or damage of the structure or to finishes, or to non-structural members (*e.g.* partition walls, claddings) is being considered, the verification for Deflection should take account of those effects of permanent and variable actions that occur after the execution of the member or finish concerned. This should be specified for each project and agreed with client.

# SERVICEABILITY LIMIT STATES FOR BUILDINGS EC3 - 2

The national Annex may specify the limits for deflection.

NOTE Guidance on which expression (6.14a) to (6.16b) to use is given in 6.5.3 and EN 1992 to EN 1999, (refer also to Course A Module 2 Slide 14)

## **Dynamic effects**

(1)B With reference to EN 1990 – Annex A1.4.4 the vibrations of structures on which the public can walk should be limited to avoid significant discomfort to users, and limits should be specified for each project and agreed with the client.

The National Annex may specify limits for vibration of floors.

# VIBRATIONS ECO

(1) To achieve satisfactory vibration behaviour of buildings and their structural members under serviceability conditions, the following aspects, amongst others,

should be considered :

a) the comfort of the user;

b) the functioning of the structure or its structural members (*e.g.* cracks in partitions, damage to cladding, sensitivity of building contents to vibrations). Other aspects should be considered for each project and agreed with the client.

(2) For the serviceability limit state of a structure or a structural member not to be

exceeded when subjected to vibrations, the natural frequency of vibrations of the

structure or structural member should be kept above appropriate values which depend upon the function of the building and the source of the vibration, and agreed with the client and/or the relevant authority.

# VIBRATIONS ECO - Continued

(3) If the natural frequency of vibrations of the structure is lower than the appropriate value, a more refined analysis of the dynamic response of the structure, including the consideration of damping, should be performed.

NOTE For further guidance, see EN 1991-1-1, EN 1991-1-4 and ISO 10137.

(4) Possible sources of vibration that should be considered include walking, synchronised movements of people, machinery, ground borne vibrations from traffic, and wind actions. These, and other sources, should be specified for each project and agreed with the client.

# **The Institution's *Manual for the design of steelwork building structures to EC35* notes the following on vibration criteria:**

- The fundamental frequency of floors in dwellings and offices should not be less than three cycles/second. This may be deemed to be satisfied when the total deflection is less than 28mm.
- The fundamental frequency of floors used for dancing and gymnasia should not be less than five cycles/second. This may be deemed to be satisfied when the total deflection is less than 10mm.



# SUPPORT ROTATION CALCULATIONS

For a uniformly distributed load  $w$  acting on a simply supported girder of effective span  $l$ , the end rotation  $\theta$  is

$$\theta = (wl^3/EI)/24 \quad (1)$$

And the mid-span deflection divided by the span length, the span deflection ratio  $\Delta/l$ , is

$$\Delta/l = (5wl^3/EI)/384 \quad (2)$$

Where  $EI$  is the flexural rigidity of the structural material.

The ratio between equations 1 and 2 works out at:

$$\theta = 3.2 \times \text{deflection-to-span ratio} \quad (3)$$

# SUPPORT ROTATION CALCULATIONS (CONTINUED)

*EC0 Basis of Structural Design* quotes rotations varying within the same limits. EN codes on bearings make various references to rotations, however specific and clear criteria may be gleaned from a PCI report<sup>3</sup>, where a simplified method catering for rotation in bearing pad design is suggested.

The allowable plastic rotation is then given at 0.015rad and 0.035rad varying on the grade of concrete and steel adopted.

# DESIGN OF BEARING PADS

The amount of bearing length required is relative to a number of considerations including span, loading and type of support. Detailed requirements of bearings take account of bearing stresses, possible spalling of support and of the supported member, and construction inaccuracies.

1/-Min. bearing related to bearing stress

2/- Spalling of masonry support 25mm

3/-Allowance for construction inaccuracies

A minimum bearing length of 75mm is specified if bearing on steelwork or concrete of minimum Grade 30, whilst on masonry this increases to 100mm. An elastomeric strip bearing has adequate rotational capacity at the support. What can however, be stated for dry pack mortar with respect to its rotational capacity?

Readings suggest that mortars in light weight material have a better rotational capacity. Tests carried out on samples indicate that these mortars work satisfactorily for semi rigid design.

# DESIGN OF BEARING PADS TO EC 2/1

The nominal length  $a$  of a simple bearing as shown in Figure 2 may be calculated as:

$$a = a_1 + a_2 + a_3 + \sqrt{\Delta a_2^2 + \Delta a_3^2}$$

where:

- $a_1$  is the net bearing length with regard to bearing stress,  $a_1 = F_{Ed} / (b_1 f_{Rd})$ , but not less than minimum values in Table 10.2
- $F_{Ed}$  is the design value of support reaction
- $b_1$  is the net bearing width, see (3)
- $f_{Rd}$  is the design value of bearing strength, see (2)
- $a_2$  is the distance assumed ineffective beyond outer end of supporting member, see Figure 10.6 and Table 10.3
- $a_3$  is the similar distance for supported member,

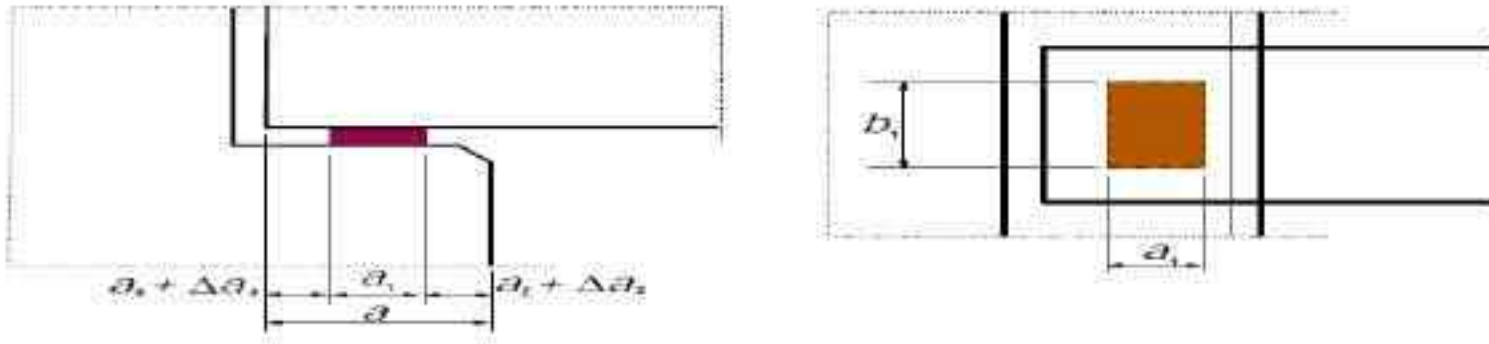


Figure 2 Example of bearing with definitions.

$f_{Rd} = 0,4 f_{cd}$  for dry connections (see 10.9.4.3 (3) for definition)

$f_{Rd} = f_{bed} \leq 0,85 f_{cd}$  for all other cases

where

$f_{cd}$  is the lower of the design strengths for supported and supporting member

$f_{bed}$  is the design strength of bedding material

# DESIGN OF BEARING PADS TO EC 2/2

**Table 1: Minimum value of  $a_1$  in mm**

Relative bearing stress, $\sigma_{Ed}/f_{cd}$	$\leq 0,15$	0,15 - 0,4	$> 0,4$
Line supports (floors, roofs)	25	30	40
Ribbed floors and purlins	55	70	80
Concentrated supports (beams)	90	110	140

**Table 2: Distance  $a_2$  (mm) assumed ineffective from outer end of supporting member. Concrete padstone should be used in cases (-)**

Support material and type	$\sigma_{Ed}/f_{cd}$	$\leq 0,15$	0,15 - 0,4	$> 0,4$
Steel	line	0	0	10
	concentrated	5	10	15
Reinforced concrete $\geq$ C30	line	5	10	15
	concentrated	10	15	25
Plain concrete and rein. concrete $<$ C30	line	10	15	25
	concentrated	20	25	35
Brickwork	line	10	15	(-)
	concentrated	20	25	(-)

**Table 3: Distance  $a_3$  (mm) assumed ineffective beyond outer end of supported member**

Detailing of reinforcement	Support	
	Line	Concentrated
Continuous bars over support (restrained or not)	0	0
Straight bars, horizontal loops, close to end of member	5	15, but not less than end cover
Tendons or straight bars exposed at end of member	5	15
Vertical loop reinforcement	15	end cover + inner radius of bending

**Table 4: Allowance  $\Delta a_2$  for deviations for the clear distance between the faces of the supports.  $l$  = span length**

Support material	$\Delta a_2$
Steel or precast concrete	$10 \leq l/200 \leq 30$ mm
Brickwork or cast in-situ concrete	$15 \leq l/200 + 5 \leq 40$ mm



job title: RESTRAINED STEEL BEAM DESIGN.

Ref.	Calculations	Outputs
reading Module 38	An end-span supported RC slab has an effective continuous span of 2m	
	Preliminary sizing $2000/27 \Rightarrow 74\text{mm} + 20\text{mm cover}$	
	Exposure XCI - interior of bldgs	
	for XCI C20/25 of $240\text{kg}/\text{m}^3$ - 25mm cover	noted C25/30 not given!
	Fire resistance R90 min thickness 100mm - min axis 30mm	
	<b>SLAB LOADING - STRUCT</b>	ECO eqn 6.10
	self-wt 0.1.25 $\Rightarrow 2.5$ $\xrightarrow{1.35}$ 3.38	
	finish 0.1.18 $\Rightarrow 1.8$ $\xrightarrow{1.35}$ 2.43	
	L.L 5 $\Rightarrow 5$ $\xrightarrow{1.5}$ 7.5	
	SLS $9.3\text{ kN}/\text{m}^2$ $\xrightarrow{1.35}$ $13.31\text{ kN}/\text{m}^2$ ULS	
$(3.38 + 2.43) + 0.7 \cdot 7.5 \Rightarrow 11.06\text{ kN}/\text{m}^2$	eq 6.10a	
$0.85(3.38 + 2.43) + 7.5 \Rightarrow 12.44\text{ kN}/\text{m}^2$	eq 6.10b	
BH $\Rightarrow 12.44 \cdot 2.5^2 / 9 \Rightarrow 8.64\text{ kN}\cdot\text{m}/\text{m}$		
The Concrete Centre spreadsheet yields		
$A_s(\text{req}) \Rightarrow 331\text{mm}^2/\text{m}$		
$A_s(\text{dev}) \Rightarrow 451\text{mm}^2/\text{m}$		
$A_s(\text{prov}) \Rightarrow 524\text{mm}^2/\text{m} \quad \times 10 @ 150\text{mm}$		
Defl check (span/250 condition)		
Permissible $L/d \Rightarrow 37.38$		
Actual $L/d \Rightarrow 35.71 \checkmark$		
* over-design due to fire - an 85mm thick slab sufficient		

job title: RESTRAINED STEEL BEAM DESIGN

Ref	Calculations	Output
	<p><u>Design of 12m steel beam - strength calculation</u></p> $M_D \rightarrow (12.44 \text{ kN/m}^2 \times 2.5 \text{ m} \times 1.15) \times 12^2 / 8 \rightarrow 643.74 \text{ kN}\cdot\text{m}$ $M_R \rightarrow f_y / z_r / \gamma_m$ $z_p \rightarrow 643.74 \times 1000 / 275 / 1 \rightarrow 2341 \text{ cm}^3$ <p><u>Deflection Check based on span/250</u></p> $I \rightarrow C_{wL}^3$ $C \rightarrow 1.24 \times 250 / 200 \rightarrow 1.55$ $W \rightarrow s_k + \psi_2 \cdot q_k$ $\rightarrow (0.125 + 0.118) + 0.6 \cdot 5 \rightarrow 7.3 \text{ kN/m}^2$ $C \rightarrow 1.55 \times (7.3 \times 2.5 \text{ m} \times 1.15) \times 12^3$ $\rightarrow 56,213 \text{ cm}^4$ <p><u>Vibration Check based <math>&gt; 3 \text{ Hz}</math> &amp; <math>\delta &lt; 28 \text{ mm}</math></u></p> $f_{th} \rightarrow \frac{\text{span}}{\delta} \rightarrow \frac{12,000}{28} \rightarrow 429$ $C \rightarrow \frac{1.24 \times 429}{200} \rightarrow 2.66$ $I \rightarrow C_{wL}^3 \rightarrow 2.66 \times (7.3 \times 2.5 \text{ m} \times 1.15) \times 12^3$ $\rightarrow 96,469 \text{ cm}^4$ <p>Actual span/delf <math>\rightarrow 429 \times 96,469 / 102,300 \rightarrow 404</math>            Actual delf <math>\rightarrow 28 \text{ mm} \times 404 / 429 \rightarrow 263 \text{ mm}</math>  <math>f_z \approx 18 / \sqrt{8} \rightarrow 18 / \sqrt{2} \times 39 \rightarrow 3.508 \text{ Hz} &gt; 3 \text{ Hz} \checkmark</math></p>	<p>IPE 500  <math>z_p \rightarrow 2613 \text{ cm}^3</math>  <math>W \rightarrow 107 \text{ kg/m}</math></p> <p>ECO            eqv 6:166</p> <p>IPE 500  <math>I \rightarrow 53,780 \text{ cm}^4</math>  <math>W \rightarrow 107 \text{ kg/m}</math></p> <p>IPE V 550  <math>I \rightarrow 102,300 \text{ cm}^4</math>  <math>t \rightarrow 216 \text{ mm}</math>  <math>W \rightarrow 159 \text{ kg/m}</math></p>



job title: RESTRAINED STEEL BEAM DESIGN

Ref	Calculations	Output
	<p><u>Support Rotation Check</u></p> <p>Rotation at support <math>3 \cdot 2 / \text{span} : \text{dotted line}</math></p> <p><math>\Rightarrow 3 \cdot 2 / 404 \Rightarrow 0.00792 \text{ rad}</math></p> <p>total rotation</p> <p><math>0.00792 + 0.005 (\text{uncertainty factor}) \Rightarrow 0.129 \text{ rad}</math></p> <p><u>BEARING LENGTH CALCULATION</u></p> <p>beam stress <math>\Rightarrow 0.4 f_{ck} / \gamma_m</math></p> <p><math>b_L \Rightarrow \sqrt{V / (0.4 f_{ck} / \gamma_m)} / b</math></p> <p><math>\Rightarrow (1244 \cdot 2.5 \cdot 1.5) \frac{12}{2} / \frac{0.4 \cdot 25}{1.5} ((0.65 \cdot 216 \text{ mm})) \Rightarrow 229 \text{ mm}</math></p> <p>allowance for tolerance <math>12 \text{ m} \times 4 \text{ mm/m} \Rightarrow 48 \text{ mm}</math></p> <p>allowance for spalled edge <u>15 mm</u></p> <p>292 mm</p> <p><u>Alternative DESIGN in Prestressed Planks</u></p> <p>SAFE LOAD: <math>1.8 \text{ kN/m}^2 + 5 \text{ kN/m}^2 \Rightarrow 6.8 \text{ kN/m}^2</math></p> <p>try a 400 mm thick panel <math>\Rightarrow 9.61 \text{ kN/m}^2</math></p> <p>compare DL at <math>6.88 \text{ kN/m}^2</math> to <math>3.38 \text{ kN/m}^2</math></p> <p>span / <math>\delta \Rightarrow 472 \Rightarrow 360</math></p> <p><math>b_L \Rightarrow 12.44 \text{ kN/m}^2 \cdot \frac{12 \text{ m}}{2} / \frac{0.4 \cdot 25}{1.5} ((0.65 \cdot 1000)) \Rightarrow 11.5 \text{ mm} &lt; 30 \text{ mm}</math></p> <p>allowance for tolerance <math>12 \text{ m} \times 4 \text{ mm/m}</math></p> <p>allowance for spalled edge</p>	<p>within acceptable range of <math>0.015 \text{ rad} - 0.035 \text{ rad}</math></p> <p><u>292 mm</u></p> <p><u>15 mm</u></p> <p>275 mm</p> <p>30 mm</p> <p>48 mm</p> <p>10 mm</p>
F01/3		88 mm < 100 mm