Technical note

Deflection and preliminary vibration effects on structural elements

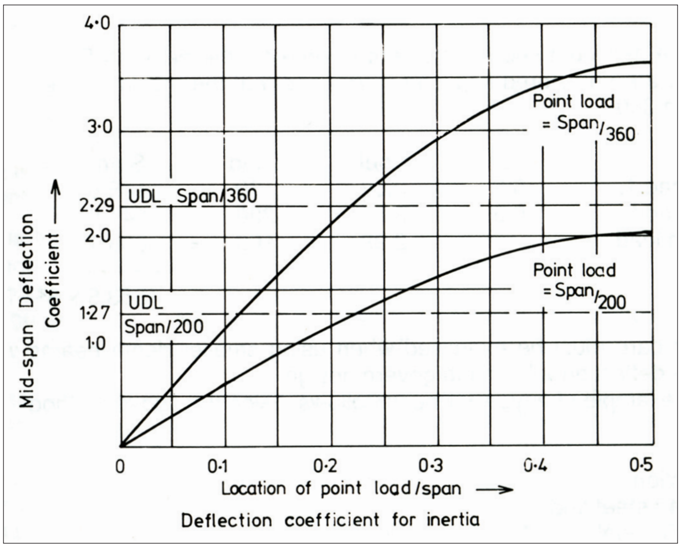
A regular contributor to Verulam, Denis Camileri consolidates his views on the subject of deflection and vibration effects in single spans

The structural strength calculations are based on safety characteristics and are well advanced. The serviceability requirements dwelling on deflection, rotation at supports and vibration characteristics depend on the use of the structure and its effect on the user. An agricultural shed can suffer deflections to a greater extent than the non-load bearing partitions in residential premises, with induced cracking that is less tolerable to the user. Impairment may also occur to the function of a building, as in the case of operating machinery.

Over the years in structural engineering span:deflection ratios in the region of 200 to 360 except for purlins and sheeting rails have been discussed in various Codes of Practice. The span:deflection ratio of 360 was specified for buildings which had non-load bearing partitions. More recently it was noted that the 360 limit had to be treated with caution, with ratios of 500 to 800 being quoted. These ratios truly depend on the rigidity of the non-load bearing partitions. Partitions of the less ductile type such as masonry or concrete blockwork possibly clad in ceramic wall tiling in bathrooms should tend towards the higher ratio. It is quite embarrassing to explain to an annoyed user that his cracking pattern is of no structural concern and that he has to learn to live with these cracks, as deflections will carry on occurring for an approximate 10y period, while drying shrinkage takes place.

The Eurocodes note that deflection limits should be specified for each project, after discussions with the client. Various National Annexes then give guidance on which ratios are to be adopted for the particular case.

How are these various span:deflection ratios to be tackled in a design office? Furthermore, it is also noted that these span:deflection ratios sometimes relate to the total deflection occurring, whilst in other instances they relate to the deflection induced solely by the imposed loading, following the completion of the construction elements. Deflection or vibration calculations can be daunting exercises in a design office.



1 Deflection coefficient for inertia *(reproduced courtesy of BCSA)*

Deflection coefficient 'C' for inertia		
Span to deflection	steel	timber E = 7000N/mm²
1/200	1.27	37.2
1/360	2.29	76.1
1/500	3.17	92.8
1/800	5.08	148.8
1/1000	6.35	186

Table 1 Updated 'C' inertia values for steel and timber

Span: deflection ratios for steelwork and timber

The clue to all this exists in a chart (Fig 1) developed in a steelwork BCSA publication, *Handbook of Structural Steelwork*. The BCSA outlines a convenient method, catering for deflection criteria in steelwork by limiting the moment of inertia of the section to: $I = CwL^3$ for central deflection when subjected to a udl, where C can be obtained from Fig 1 for a udl or point loads.

Where I is in cm^4, L is in metres and w is in kN/m.

This chart can easily be updated for any required span:deflection ratio required, as any value is directly obtained by scaling. Other materials, for example structural timber, may be related to this by topping the C value in proportion to the E values of the respective materials $C_{timber} = C_{steel} \times E_{st}/E_{tim}$.

Table 1 updates the C values for the more onerous span to deflection limits, whilst the final column gives the coefficient C for timber sections with E = 7000N/mm². This again is obtained directly by scaling the C value for the steelwork E value taken at 205 000N/mm².

For a continuous span the above C values may be reduced by 0.7 for a two span combination and by 0.52 for a three span combination for a udl loading, whilst factors of 0.72 and 0.55 respectively may be used, for the case of central point loads (National Cooperative Highway Research Program (NCHRP)).

Span: depth ratios for concrete structures

In the case of reinforced concrete the span:deflection ratio is taken over by span-to-depth ratios. The span-to-depth ratio of 20 for simply supported spans is based on a span-to-deflection ratio of 1:250.

From bending theory:

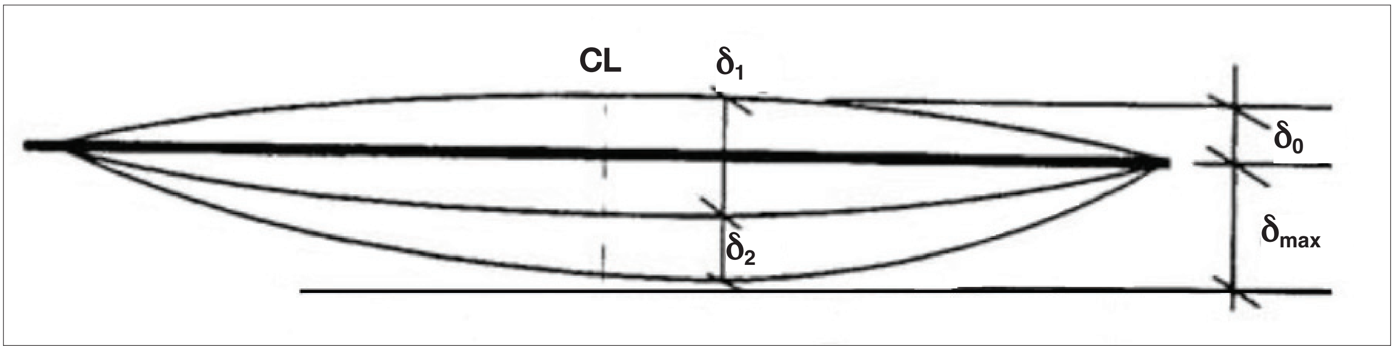
Span/depth $L/d = 4.8 \times E/(f_{cu} \times q)$

Where q is the allowable span to deflection factor, which for 1/500 works out at:

Span/depth = 4.8 × 28kN/mm²/(25N/mm² × 500) = 10.75

This as compared to the conventional 20 specified in BS 8100.

Through consultation of a 1974 publication, *Reinforced concrete*



2 Deflection limits: δ_0 = deflection due to pre-camber; δ_1 = deflection due to dead load; δ_2 = deflection due to live load

design to CP110 – simply explained by A. H. Allen it was noted that the span-to-depth ratios are based on a final deflection not exceeding span/250, with the final deflection to partitions and finishes that takes place after construction is completed, limited to span/350 or 20mm whichever is the lesser. Caution is then emphasised about damage to finishes and partitions, as abiding by the span/350 or 20mm requirement does not guarantee that partitions will be undamaged by deflection. It then goes on to suggest that block walls may be seriously cracked by deflections of the order of span/800 or less.

BS 8110 Part 2 states that deflection is noticeable if it exceeds L/250, with deflection due to dead load being possibly offset by pre-cambering. The standard then states that damage to partitions, cladding and finishes will generally occur if the deflection exceeds L/500 or 20mm whichever is the lesser for brittle finishes, with L/350 or 20mm whichever is the lesser for non-brittle finishes.

EC2 states that the appearance and general utility of a structure may be impaired when the calculated sag of a beam, slab or cantilever subject to quasi-permanent loads exceeds span/250. Where partitions are in contact with or attached to members, it may be necessary to limit the deflection after construction to span/500. Further, for spans other than flat slabs exceeding 7m, supporting partitions liable to be damaged by excessive deflections, the span/depth ratio should be multiplied by 7/span, whilst in BS 8110 this applies for spans in excess of 10m.

Should not a more 'hands on' method exist, whereby different span/depth ratios are quoted for deflections limited for various ratios as listed above? The Concrete Centre element design Eurocode spreadsheet contains a tab for when brittle partitions are supported on a slab. Could this tab not be updated to cater for a variety of span:deflection ratios giving the facility to the designer to ascertain which ratio is suitable for a particular project?

The structural engineer will then be provided with the facility to decide on how much lower should the span/depth ratios be than the stipulated 20 or 26 to limit cracking to partitions. Even if one is designing a roof slab to a shed how much higher than a span to depth ratio of 20 for a simply supported span may one go without suffering from the serviceability aspect?

For pre-stressed slab panels span to depth ratios are specified varying from 30 down to 40 in the span range of 6.0m up to 13.0m. The high span to depth ratio refers to an office LL (live load), with the low value referring to warehouse loading.

Table 2 refers to hollow core pre-stressed panels utilised as transfer slabs, supporting overlying masonry constructions for up to four to five storeys on a span of 6.50m. To be noted that the deflection/span ratio varies from 307 up to 667, generally above the 360 limits mentioned above, explaining possibly the reason for minimal cracking to this form of construction. The span-to-depth ratios quoted in Table 2 varying from 26 down to 13, are more closely related to beam than slab sections.

The PCI Manual for hollow core pre-stressed slabs, further specifies that the span:depth ratios of flat roofs may be limited to 1:180 for the LL (live load) applied, δ_2 in Fig 2, although this limit is not intended to safeguard against ponding.

Long span lightly loaded pre-stressed slabs on the other hand are subjected to camber. These on a 13m span could be subjected to camber/span ratios in the region of 200, which could aid in the shedding of rainwater from canopy constructions.

Slab depth (mm)	Safe loading (kN/m)	Span/Total deflection ratio (δ_{max}, Fig 2)	Span: depth ratio
250	15	1:307	26.00
330	30	1:395	19.70
450	55	1:588	14.45
500	72.5	1:667	13.00

Table 2 Prestressed transfer panels deflection characteristics

Vibration to EC3 (steelwork) and EC5 (timber)

Structural engineers are normally static in their calculations, however situations do arise where a more dynamic approach is required to limit nuisance from vibrations. The Institution's *Manual for the design of steelwork building structures to EC3* notes:

– The fundamental frequency of floors in dwellings and offices (EC3 – steelwork) should not be less than 3 cycles/s. This may be deemed to be satisfied when:
  $\delta_1 + \delta_2 < 28$mm (see Fig 2)
– The fundamental frequency of floors used for dancing and gymnasia (EC3 – steelwork) should not be less than 5 cycles/s. This may be deemed to be satisfied when:
  $\delta_1 + \delta_2 < 10$mm. (see Fig 2)
– For domestic timber floors (EC5 – timber), the fundamental frequency is to lie between 8Hz<f<40Hz. This may be deemed to be satisfied when:
  $\delta_1 + \delta_2 < 14$mm (see Fig 2)

Thus, it is noted that a complex vibration calculation has been converted into structural engineers' parlance using a deflection computation, with the above deflection aids being of guidance.

The Institution's manual then notes that the deflection calculation must comply with several limits and then continues on to quote the *C* value at 6.2, which from Table 1 notes a span to deflection ratio of 1:1000, which is far removed from the established 1:250 to 1:360 often quoted ratios. This blanket *C* value apparently also caters for vibration effects, when considered necessary.

Recommendation

The above comments attempt to introduce flexibility in applying the various span-to-deflection or span-to-depth ratios used with the various structural materials encountered on projects. The simple procedure outlined in this technical note will guide the structural engineer in conforming to the Eurocode requirement, that deflection limits should be specified for each project, after discussions with the client. The structural engineer is also in a better position to differentiate between situations depending solely on deflection criteria or whether vibration nuisance should also be considered.