

Deflection and preliminary vibration

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The structural strength calculations being well advanced are based on safety characteristics. The serviceability requirements dwelling on deflection, rotation at supports and vibration characteristics depend on the use of the structure and its effect on the user. An agricultural shed can suffer deflections to a greater extent than the non-loading bearing partitions in a residential premises, with induced cracking that is less tolerable to the user. Impairment may also occur to the function of a building, in the case of operating machinery.

Over the years in Structural Engineering, span:deflection ratios in the region of 200 to 360, except for purlins and sheeting rails, have been discussed in various Codes of Practice. The span:deflection ratio of 360 was specified for buildings which had non-load bearing partitions. Later on it was noted that the 360 limit had to be treated with caution, with ratios of 500 to 800 being quoted. These ratios truly depend on the rigidity of the non-loading bearing partitions. Partitions of the less ductile type such as masonry or concrete blockwork possibly clad in ceramic wall tiling in bathrooms should tend towards the higher ratio. It is quite embarrassing to explain to an annoyed user that his cracking pattern is of no structural concern and that he has to learn

to live with these cracks, as deflections will carry on occurring for an approximate 10 year period, due to the existent drying shrinkage occurring.

The Eurocodes note that deflection limits should be specified for each project, after having been discussed with the client. Various National Annexes then give guidance on which ratios are to be adopted for the particular case.

How are these various span:deflection ratios to be tackled in a design office? To be further noted that these span:deflection ratios sometimes relate to the total deflection occurring, whilst on other instances relate to the deflection induced solely by the imposed loading only, following the completion of the construction elements. Figure 2 outlines the various deflections occurring over time. Deflection or vibration calculations may be daunting exercises in a design office.

SPAN: DEFLECTION RATIOS FOR STEELWORK & TIMBER

The clue to all this luckily exists in Figure 1 developed in a steelwork BCSA publication, Handbook of Structural Steelwork. The BCSA outlines a convenient method, catering for deflection criteria in steelwork by limiting the moment of inertia of the section in cm^4 to: $I = CwL^3$ for central deflection as subjected to an uniformly distributed load (UDL), where

Table 1:

Deflection coefficient 'C' for inertia		
Span to deflection	Steel	Timber $E = 7000\text{N/mm}^2$
1/200	1.27	37.2
1/360	2.29	67.1
1/500	3.17	92.8
1/800	5.08	148.8
1/1000	6.35	186

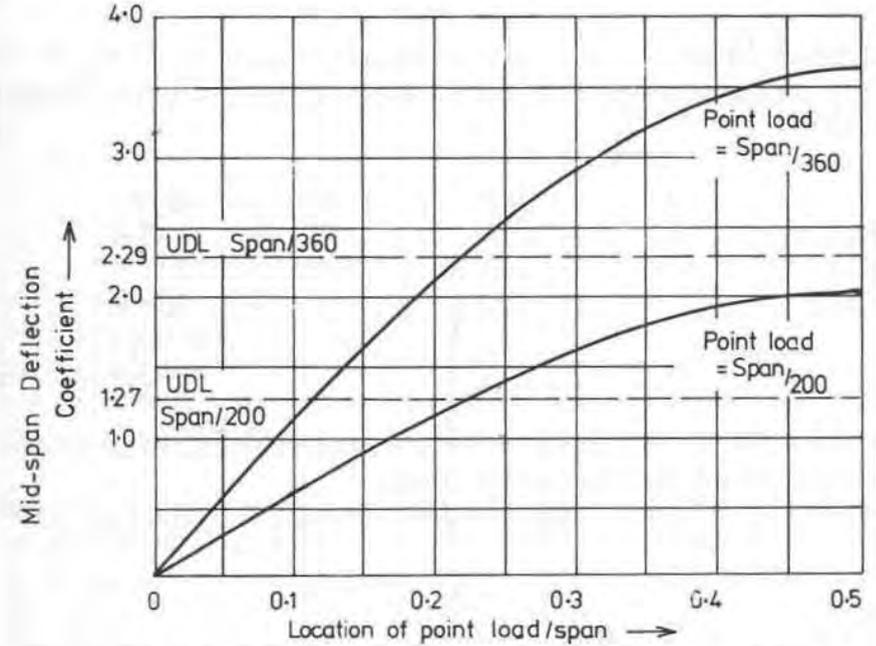


Figure 1 Deflection coefficient for inertia Source: BCSA

C can be obtained from Figure 1 below for a UDL or point loads; L is in metres; and w is in KN/m .

This chart can easily be updated for any required span:deflection ratio required, as any value is directly obtained by scaling. Other materials, for example structural timber, may be related to this by topping the C value in proportion to the E values of the respective materials, mainly steelwork and timber.

Table 1 updates the C values for the more onerous span to deflection limits, whilst the final column gives the coefficient C for timber sections with $E = 7,000\text{N/mm}^2$. This again is obtained directly by scaling the C value for the steelwork E value taken at $205,000\text{N/mm}^2$. For a continuous span the above C values may be reduced by 0.7 for a 2 span combination and by 0.52 for a 3 span combination for a UDL loading, whilst factors of 0.72 and 0.55 respectively, for the case of central point loads (National Cooperative Highway Research Program (NCHRP)).

SPAN: DEPTH RATIOS FOR CONCRETE STRUCTURES

In the case of reinforced concrete, the span:deflection ratio is taken over by span to depth ratios. The span-to-depth ratios of

20 for simply supported spans is based on a span-to-deflection ratio of 1:250.

From bending theory:

$$\text{Span/depth: } L/d = 4.8 * E / (f_{cu} * q)$$

Where q is the allowable span to deflection factor, which for 1/500 works out at: $\text{Span/depth} = 4.8 * 28 \text{ kN/mm}^2 / (25\text{N/mm}^2 * 500) = 10.75$

This as compared to the conventional 20 specified in BS 8100.

Leafing through a 1974 publication, Reinforced concrete design to CP110 – simply explained by A. H. Allen the following was noted. The span-to-depth ratios are based on a final deflection not to exceed span/250, with however the final deflection to partitions and finishes that takes place after construction is completed, limited to span/350 or 20mm whichever is the lesser. Caution is then emphasized about damage to finishes and partitions, as abiding by the span/350 or 20mm requirement does not guarantee that partitions will be undamaged by deflection. It then goes on to suggest that block walls may be seriously cracked by deflections of the order of span/800 or less.

BS 8110 Pt2 states that deflection is noticeable if it exceeds $L/250$, with deflection due

on effects on Structural Elements

to dead load being possibly offset by pre-cambering. The code then states that damage to partitions, cladding and finishes will generally occur if the deflection exceeds $L/500$ or 20mm whichever is the lesser for brittle finishes, with $L/350$ or 20mm whichever is the lesser for non-brittle finishes.

EC 2 states that the appearance and general utility of a structure may be impaired when the calculated sag of a beam, slab or cantilever subject to quasi-permanent loads exceeds $span/250$. Where partitions are in contact with or attached to members, it may be necessary to limit the deflection after construction to $span/500$. Further, for spans other than flat slabs exceeding 7m, supporting partitions liable to be damaged by excessive deflections, the span/depth ratio should be multiplied by $7/span$, whilst in BS 8110 this applies for spans in excess of 10m.

Should not a more 'hands on' method exist, whereby different span/depth ratios are quoted for deflections limited for various ratios as listed above? The Concrete Centre element design Eurocode spreadsheet contains a tab for when brittle partitions are supported on a slab. Could this tab not be updated to cater for a variety of span:deflection ratios giving the facility to the designer to ascertain which ratio is suitable for a particular project?

The structural engineer will then be provided with the facility to decide on how much lower should the span/depth ratios be than the stipulated 20 or 26 to limit cracking to partitions. Even if a roof slab to a shed is to be designed how much higher than a span to depth ratio of 20 for a simply supported span, may one go without suffering from the serviceability aspect?

Table 2: Prestressed transfer planks deflection characteristics

Slab depth mm	Safe Loading kN/m^2	Span / Total Deflection ratio (δ_{ma} , figure 2)	Span: Depth ratio
250	15	1:533	26.00
330	20	1:1155	19.70
450	55	1:1293	14.45
500	65	1:1352	13.00

For prestressed slab panels span to depth ratios are specified varying from 30 down to 40 in the span range of 6.0m up to 13.0m. The high span to depth ratio refers to an office LL, with the low value referring to warehouse loading.

Table 2 refers to hollow core prestressed panels utilised as transfer slabs, supporting overlying masonry constructions for up to 4 to 5 stories on a span of 6.50m. To be noted that the deflection/span ratio varies from 307 up to 667, generally above the 360 limits mentioned above, explaining possibly the reason for minimal cracking to this form of construction. The span to depth ratios quoted in Table 2 varying from 26 down to 13, are more closely related to beam than slab sections.

The PCI Manual for hollow core prestressed slabs, further specifies that the span:depth ratios of flat roofs may be limited to 1:180 for the LL applied noted as δ_2 in Figure 2. This limit is not intended to safeguard against ponding.

Long span lightly loaded prestressed slabs on the other hand are subjected to camber. These on a 13m span could be subjected to camber/span ratios in the region of 200,

which could aid in the shedding of rainwater for canopy constructions.

Another domain were span to depth ratios are of importance is where precast prestressed hollow slabs are supported on non-rigid supports. Reduction of shear resistance to these precast slabs, which tests show to be in the region of 40 -77%, is due to the transverse deformation of the slab ends resulting from the deflection of the supporting beam. These beam deflections were noted to vary typically from $L/1000$ - $L/300$ (Pajari M. & Koukari H., "Shear resistance of PHC slabs supported on beams. I tests", Journal of Structural Engineering, Vol. 124, No 9,1998).

VIBRATION TO EC3 (STEELWORK) & EC5 (TIMBER)

Structural engineers are normally static in their calculations, however situations do arise where a more dynamic approach is required to limit nuisance from vibrations. The ISTRUCTE "Manual for the design of steelwork building structures to EC3" notes the following:

- The fundamental frequency of floors in dwellings and offices (EC3 - steelwork) should not be less than 3 cycles/second. This may be deemed to be satisfied when $\delta_1 + \delta_2 < 28mm$ (see Figure 2).
- The fundamental frequency of floors used for dancing and gymnasia (EC3 - steelwork) should not be less than 5 cycles/second. This may be deemed to be satisfied when $\delta_1 + \delta_2 < 10mm$ (see Figure 2).
- For domestic timber floors (EC5 - timber), the fundamental frequency is to lie between $8Hz < f < 40Hz$. This may be

deemed to be satisfied when $\delta_1 + \delta_2 < 14mm$ (see Figure 2).

Thus, it is noted that a complex vibration calculation has been converted into Structural Engineers' parlance being a deflection computation, with the deflection aids being of guidance.

The above manual then notes that the deflection calculation must comply with several limits. It then quotes the C value at 6.2, which from Table 1 notes a span to deflection ratio of 1:1,000, a far cry from the established 1:250 to 1:360 oft quoted ratios. This blanket C value apparently also caters for vibration effects, when considered necessary.

RECOMMENDATION

The above attempts to introduce flexibility in applying the various span to deflection or span to depth ratios used with the various structural materials encountered on projects. The simple procedure outlined above will guide the structural engineer in conforming to the Eurocode requirement, that deflection limits should be specified for each project, after having been discussed with the client. People in offices or residences do not like distinctly perceptible vibration, whereas people taking part in a non-stationary activity, such as dining beside a dance floor or lifting weights in an aerobic gym will accept vibrations approximately 10 times greater. Finally, the age of the building also comes into the equation. The older the building the less sensitive is its user to its existing cracks, deflections and vibration effects. The structural engineer is also in a better position to differentiate between situations depending solely on deflection criteria or whether vibration nuisance is also to be considered.

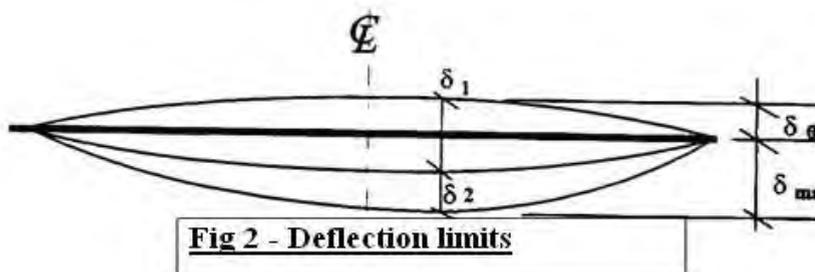


Fig 2 - Deflection limits

δ_0 = deflection due to pre-camber
 δ_1 = deflection due to dead load
 δ_2 = deflection due to live load