The author’s previous technical note1 dwelt on deflection and vibration effects as these relate to the building inconveniences caused. Various ratios were then discussed, outlining methods that may easily be adopted in design offices to engage on a particular ratio considered to cause least disturbance for the use/s anticipated.

This note demonstrates that it is an even easier step to go from a deflection span ratio onto a rotation calculation, as a simple relation connects these criteria.

Materials used for bearing pads include:

– Chloroprene (neoprene); probably the most popular
– Laminated elastomeric pads with reinforcing layers of steel or fibreglass, generally used for bridge bearings
– Steel plates mostly used as shims
– Low friction material such as Teflon® or plastic membrane strips are used to provide a slip surface, mostly used under hollow core slabs

A number of materials such as bituminous joint filler, hardboard, wood and similar filler materials have been used, but these are not considered as structural materials.

Reference to rotations in standards

If deflection and vibratory effects are engaged at some stage of the design process, less and less is catered for in the rotations arising at the bearings of structural elements. Details may be found in the CIRIA Technical Note2 which refers to rotations and quotes quite small rotations or changes of section at the supports (such as occur in reinforced concrete because of cracking) cause relatively large increases in deflection. It is then suggested that sliding and movement joints may thus be necessary to avoid cracking and rotation for brickwork construction. A movement detail for a precast element then notes that a shortening in bearing length, further compounded by translation due to rotation could lead to serious shortcomings. However, further to a suggestion not to rely on full fixity in calculating deflections no further concrete guidance is given on allowable rotations, just prudence advocated.

BS 5400-9:1983 relating to Bridge Bearings refers to rotational limitations as outlined in the following expressions:

For plain pad and laminated bearings, the total vertical deflection, \( D \), should satisfy the expression

\[ \Delta M_{Ed} = F_{Ed,sup} t / 8 \]

Where:

- \( F_{Ed,sup} \) is the design support reaction and \( t \) is the breath of the support.

\( \Delta M_{Ed} \) is the angle of rotation (if any) across the length \( l \), of the bearing (in radians)

It is further stated that plane-sliding bearings normally provide for translation only. Rotation can be permitted in accordance with flat sliding surfaces and should not be used to accommodate rotation other than about an axis perpendicular to the plane of sliding. Other provisions should be made for rotation about an axis in the plane of sliding. BS 8110 notes that when large rotations occur, without defining what is large, suitable bearings should be used. Reference is then made that these rotations may throw the line of action onto the outer face, necessitating larger bearing stresses. EC2 refers to where a beam or slab is continuous over a support which may be considered to provide no restraint to rotation (e.g. over walls), the design support moment, calculated on the basis of a span equal to the centre-to-centre distance between supports, may be reduced by an amount as follows:

\[ \Delta M_{Ed} = F_{Ed,sup} t / 8 \]

Where:

- \( F_{Ed,sup} \) is the design support reaction and \( t \) is the breath of the support.

The allowable plastic rotation is then given at 0.015rad and 0.035rad varying on the grade of concrete and steel adopted. Again EC2 Basis of Structural Design quotes rotations varying within the same limits. EN codes on bearings make various references to rotations, however specific and clear criteria may be gleaned from a PCI report3, where a simplified method catering for rotation in bearing pad design is suggested. The report noted that most codes related to large bridge bearings with small pads

**Technical note**

**Rotation of supporting small bearing pads in buildings**

Denis Camilleri of DHI Periti, Malta provides a follow-up to his previously published technical note (Vol.89 Iss.5), this time on the subject of calculating the rotational forces on bearing pads.

![Strain vs shear relationship for bearing pads of different hardness](https://example.com/image1)

\( \alpha_b \) is the angle of rotation (if any) across the length \( l \), of the bearing (in radians)

1. Strain vs shear relationship for bearing pads of different hardness (Source: Kim S. Elliott, Precast concrete structures,© Elsevier)
Table 1  Prestressed transfer planks deflection/rotation characteristics

<table>
<thead>
<tr>
<th>Slab depth</th>
<th>Safe loading kN/m²</th>
<th>Span/Total deflection ratio</th>
<th>Span/Depth ratio</th>
<th>Rotation at support at radian</th>
</tr>
</thead>
<tbody>
<tr>
<td>250</td>
<td>35</td>
<td>1.357</td>
<td>0.86</td>
<td>0.0104</td>
</tr>
<tr>
<td>330</td>
<td>30</td>
<td>1.585</td>
<td>0.86</td>
<td>0.0081</td>
</tr>
<tr>
<td>450</td>
<td>20</td>
<td>1.568</td>
<td>1.45</td>
<td>0.0054</td>
</tr>
<tr>
<td>500</td>
<td>25</td>
<td>1.667</td>
<td>13.30</td>
<td>0.0048</td>
</tr>
</tbody>
</table>

Table 2  Deformation of bearing pads under varying climatic conditions

<table>
<thead>
<tr>
<th>Slab depth</th>
<th>Deformation Due to compressive load – mm (A)</th>
<th>Due to rotation (rad) – internal environment – mm (B)</th>
<th>Maximum deformation at loading – mm (A + B)</th>
<th>Minimum deformation at rear edge of loading – mm (A – B)</th>
</tr>
</thead>
<tbody>
<tr>
<td>500mm deep</td>
<td>0.05</td>
<td>0.45</td>
<td>1.05</td>
<td>0.6</td>
</tr>
<tr>
<td>500mm deep</td>
<td>0.05</td>
<td>1.125</td>
<td>2.175</td>
<td>-0.075</td>
</tr>
<tr>
<td>500mm deep</td>
<td>0.05</td>
<td>0.56</td>
<td>0.86</td>
<td>-0.26</td>
</tr>
<tr>
<td>500mm deep</td>
<td>0.05</td>
<td>1.35</td>
<td>1.65</td>
<td>-1.05</td>
</tr>
</tbody>
</table>

Table 1  Prestressed transfer planks deflection/rotation characteristics

required in buildings regulated to a secondary position. The objectives of the report were to develop adequate design criteria for precast concrete buildings. A common rule of thumb is quoted as:

\[ R = \frac{2Dc}{L \quad \ldots 4} \]

Where: \( R \) rotation is given in radians, \( Dc \) is the compression displacement, \( L \) is the length of bearing in direction of rotation.

On the assumption that a minimum displacement of 0.15t occurs under design load, the above rule of thumb equation becomes maximum rotation in radians: \( R < \frac{0.21}{L} \quad \ldots 5 \)

Where: \( t \) is the pad thickness, \( L \) is the direction of the pad taken in either of one of the principal dimensions where the rotation occurs.

This PCI publication further recommends that the length and width of the unreinforced pad should be > 5 thickness for stability. The thickness \( t \) should be > 6mm for stemmed members and >10 mm for beams. Unreinforced pads with shape factor \( S < 2 \) should be avoided for “bees” and \( S < 3 \) avoided under beams.

The shape factor \( S \) is a means of taking account of the shape of the elastomeric layer in strength and deflection calculations. It is obtained from reference literature and the expression:

\[ S = \frac{A}{(2L/t)} \quad \ldots 6 \]

For strip bearings the shape factor \( S \) is given by the expression:

\[ S = \frac{A}{(2L/t)} \quad \ldots 7 \]

Where:

\( A \) = the overall plan area of the elastomeric bearing
\( t \) = the thickness of the bearing material
\( L = \) the pad length
\( t < \) the effective thickness of an individual elastomeric layer in compression

The shape factor defines how thin the layer is compared with its lateral dimensions. For an infinitely wide strip bearing, \( W \) is infinite and \( S = \frac{A}{L/2t} \).

For a square, \( S = \frac{L}{4t} \), ...9

and for other rectangular shapes, \( S \) lies between these two bounds.

Common beams have \( S \) in the range 3 < \( S < 8 \). The shape factor also provides a useful basis for normalizing the compressive stress, \( \sigma \), since the shear strain caused by compression is, according to small displacement theory, directly proportional to \( \epsilon \times G \), where \( G \) is the shear modulus of the pad material.

Increasing the shape factor \( S \) therefore increases the axial stiffness and strength, but it reduces the ability of the bearing to accommodate rotation. These opposite tendencies may cause a dilemma in design. A larger bearing with a higher shape factor would carry the axial load better, but it would reduce the bearing’s ability to accommodate rotations. It is worth noting that such design involves the use of a mixture of force and displacement loadings and that this combination presents challenges. The axial load is a force yet the rotation is a displacement. Designing for both simultaneously requires that the bearing be stiff in compression yet flexible in rotation. That may be difficult, because the features (size, shape factor) that make it stiff in compression tend also to make it stiff in rotation.

Support rotation calculations

For a uniformly distributed load \( w \) acting on a simply supported girder of effective span \( l \), the end rotation \( \theta \) is

\[ \theta = \frac{w l^3}{6EI} \quad \ldots 10 \]

And the mid-span deflection divided by the span length, the span deflection ratio \( \Delta /l \), is

\[ \Delta /l = \frac{5wl^3}{384EI} \quad \ldots 11 \]

Where \( E \) and \( I \) are the flexural rigidity of the structural material.

The ratio between equations 10 and 11 works out at:

\[ \frac{\Delta}{\theta} = \frac{1}{24} \quad \frac{1}{1384} \quad \frac{3.2}{3.2} \quad \ldots 12 \]

Similar calculations for a single concentrated load at mid-span give a ratio of 3.0. The end rotation consistent with a udl deflection of 1800, is given at:

\[ \frac{\Delta}{\theta} = \frac{2}{3} \quad \frac{800}{0.0049} \]

The end rotation for any other deflection limit can be obtained directly by scaling.

For continuity over two or three spans, the mid-span deflections and the end rotations (at the central support) are multiplied by factors as obtained from reference literature. Because in multi-spans the end rotation decreases more than the mid-span deflection, the net effect is to reduce the end rotation if the mid-span deflection is still controlled by the udl limit. If the entire load is treated as distributed, the largest possible end rotation is

\[ 0.50/0.7 \quad 0.004 = 0.0038 \text{ radians for two spans, and} \quad 0.40/0.52 \quad 0.004 = 0.0030 \text{ radians for three spans.} \]

Calculations then compile an (amplified) rotation comprising:

Allowance for uncertainty: 0.005
Thermal camber: 0.0015
Loading: 0.0040
Total: 0.0105 rad

In particular, the allowance for uncertainty is a very small angle given as 0.005 radians. This corresponds to a movement from center of about one tenth of the bubble length in a carpenter’s...
As a minimum bearing length of 75mm is specified if bearing on steelwork or concrete of minimum Grade 30, whilst on masonry this is given at 100mm. Thus, this bearing length of 100mm is to be adopted. Similarly, for the 500mm deep section the bearing length works out at 110mm. The above minimum bearing lengths give an idea of where a steel flange not less than 180mm is desired for supporting precast slabs on both sides.

For a Shore hardness 60 elastomeric strip bearing pad of 90mm depth and 6mm thickness: Shape factor S as per equation 7 given at: 90/2 x 6 = 7.5 compressive stress on pad for the 500mm unit on a 6m span works out at: 72.5kN/m x 3m / 0.09m = 2.4kN/mm².

For this compressive stress, Fig 1 outlines a strain as imposed on the elastomeric pad of 0.175. This imposes a compression (column A Table 2) on the bearing pad of the 500mm deep unit given at: 0.175 x 6mm = 1.05mm

Similarly for the 250mm unit the compressive stress works out at: 15kN/m x 3m / 0.075m = 6.0kN/mm².

For this compressive stress, Fig 1 outlines a strain as imposed on the elastomeric pad of 0.05. This imposes a compression (column A Table 2) on the bearing pad of the 250mm deep unit given at: 0.05 x 6mm = 0.3mm

PCI equation 5 limits rotation to: 0.3 x 6 /90 = 0.02rad.

This is less than the rotations quoted above at 0.0248 and 0.0304 rad for the above precast 500mm and 250mm deep sections. On the other hand, if these precast units are internal units not subjected to thermal strains, then the respective rotations are reduced to 0.017 and 0.015rad respectively and considered acceptable.

Where the rotation θ is known the deformation Δt = ±0.5b

2  Behavior of elastomeric bearing pads. (Source: Kim S. Elliott, PreCast concrete structures. © Elsevier)
Epoxy mortars are known to develop compressive strengths after 28 days of 90N/mm² and have been adopted as bridge bearings. However, possibly such epoxy materials will be too rigid to take the support rotations imposed. Repair mortars with a build up of fibre-reinforced powder mix and modified styrene butadiene latex gauging liquid gives a 28 day strength of 22.5N/mm². This may be considered more flexible, noting as above that, the bearing stresses under direct load is achieved at 2.4N/mm². Should lightweight repair mortars (as suggested in the paper) be taken into consideration?

Conclusions and recommendation
An easy relationship exists, noted in equation 12, to convert span to deflection ratios by a constant 3.2 to rotations in radians. However, it is noted that unlike major bridge structures, relatively little importance is given to the rotation bearing seating capacity for buildings. The effect induced by rotations was noted to increase for exposed structures, with increased rotations noted for lighter sections, even though subjected to much lighter loadings. This as evidenced from results noted in Table 2.

Crack patterns have been known to exist for roof structures and possibly attributed to thermal and shrinkage cracking. The placing of a plastic sheet material prior to the casting of a concrete slab has been advocated in order to mitigate crack formation. This detail has not always been successful and the above demonstrates that there exists more than just thermal movement, with rotation at the support being the major cause for this cracking.

Possibly the dry pack mortars that are being specified as seating bearings are too rigid, not providing sufficient flexibility for rotation to occur. This in turn relates to spalling of the supporting section, eventually resulting on exposing the reinforcement. The deterioration then sets in, necessitating structural repair jobs. More research and testing on the specification of a dry pack mortar, together with outlining the shape of the resulting bearing stress block are presently called for.

References
1 Camilleri, D: Technical Note 'Deflection and preliminary vibration effects on structural elements' The Structural Engineer, 1 March 2011, 89/3, p 17/18
2 'Design for Movement in Buildings', CIRIA Technical Note 107, 1981
4 'Rotation Limits for Elastomeric Bearings', National Cooperative Highway Research Program (NCHRP) report 596, 2008
5 Technical Note, Precast Flooring Federation, 2005